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A Theory of City Size

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constant van der Waals background force between the tip and the metal substrate, also simplifying data interpretation. Recently, Gross *et al.* were even able to determine the bond orders within cyclic hydrocarbons and fullerenes with force microscopy (*8*).

When elements such as nitrogen and oxygen are present, as in the case of cephalandole A (*9*), image interpretation becomes more complicated, because whereas the carbon monoxide tip is repelled by carbon and hydrogen, it is attracted to nitrogen and oxygen. Atomic identification of single chemical species by force spectroscopy, pioneered by Sugimoto *et al.* (*10*) for semiconductor surfaces, might become feasible for the chemical analysis of individual atoms in single molecules. This would allow the study of reactions of more complex individual molecules.

The study of molecules that are not flat is also very challenging, because force microscopy senses short- and long-range forces (*6*), and long-range forces change dramatically with distance. Recently, Repp and coworkers successfully imaged dibenzo[a,h] thianthrene, a butterfly-shaped molecule with the body flat on the surface and the two wings reaching up from the surface (*11*). The authors brought the oscillating tip very carefully to the molecule and used a combined method of tunneling current feedback while recording the frequency shift for determining the adsorption site. To extend the method to even more challenging three-dimensional molecules such as DNA, not only the tip-terminating carbon monoxide molecule at the front end would be important but the overall sharpness and crystallographic orientation of the tip would need to be known (*12*).

The observation of chemical reactions by force microscopy at submolecular resolution reported by de Oteyza *et al.* is of great value in cases where a reaction results in an ensemble of products, as in the decay of phenylene-1,2-ethynylene. It also appears possible to induce the formation of chemical bonds by forces exerted by the tip of the atomic force microscope, similar to reactions induced by tunneling currents (*13*) or the flipping of atom dimers on a semiconductor surface (*14*). Twenty-seven years after Binnig, Quate, and Gerber introduced the atomic force microscope (*15*), it remains exciting to watch what this technological marvel holds in store for the future.

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SOCIOLOGY

A Theory of City Size

A theory explaining how the attributes of cities scale with city size may help to inform urban planning.

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As cities get bigger, they generate economies and diseconomies of scale, referred to by Marshall more than a century ago as the effects of agglomeration (*1*). Simple theories assume that cities exist due to a trade-off between these positive and negative forces of agglomeration and that the benefits continue to outweigh the costs of cities as they grow ever larger (*2*). But precisely what happens as cities grow? On page 1438 of this issue, Bettencourt (*3*) shows that the ways in which people interact with one another in cities lead to power laws—or allometric laws—that relate population, area, and attributes of the population to scale.

Bettencourt *et al.* have previously demonstrated the qualitative changes associated with the scale of cities (*4*). The larger a city, the greater the benefits with respect to attributes such as income earned, but also the greater the costs with respect to social interactions such as crime (*4*). There are also gains in efficiency of infrastructure provision, because as cities get larger, they use less space per capita for utilities, transport routes, and residential living. These allometric laws are demonstrable for socioeconomic attributes such as income, the production of patents, financial services, and crime, all of which scale superlinearly with respect to population.

There is substantial evidence for these scaling relationships in countries such as in the United States, where cities are widely spaced and do not generally merge into one another. It is even consistent for countries that have dominant, or so-called single primate cities unrivaled by others, such as London, which tend to be outliers in city systems that do not manifest clear scaling. Bettencourt develops a theory based on a social dynamic that accounts for these observations. His argument turns on associating the trade-off between the positive feedback associated with social interactions generating more than proportionate increases in resources and the negative feedbacks resulting from mixing of populations over ever larger areas as cities get bigger. These negative feedbacks include those associated with travel and congestion.

This simple but appealing and plausible idea resonates with what we know about cities. The power of Bettencourt's analysis is to show how the trade-off between benefits and costs leads to increases in quantities that vary superlinearly with population size, such as income and resources, and infrastructures that vary sublinearly, such as the areas and lengths of physical links such as streets. He disentangles this theory by associating the ways in which we build infrastructures incrementally as treelike networks, showing that his derivations of scaling relations can be related to the underlying fractal geometry of the cities under scrutiny (*5*).

Bettencourt develops a recursive formulation for this geometry, using analogies with electric circuits to show how energy dissipates and scales like human interactions, with the ratio of income to total power used being a constant with respect to city size. He then reworks this framework using a maximization of the differences between income and energy with respect to average social outputs subject to known constraints. This implies the same kinds of scaling that are introduced in his initial discussion of

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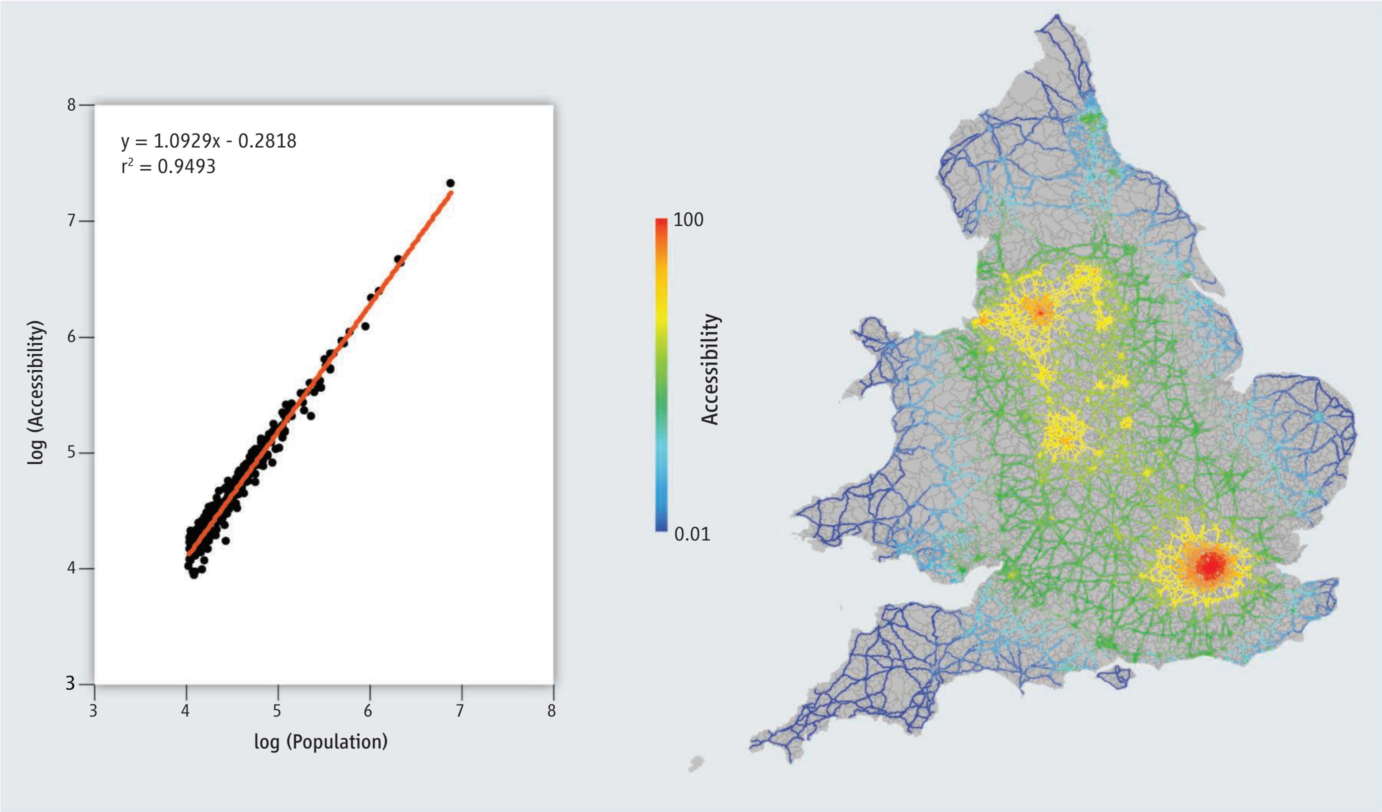

Superlinear scaling of accessibility for cities in England and Wales. Accessibilities to all 8850 small areas (wards) from 283 towns and cities with a population of more than 10,000 are computed over the road network, which is colored according to the accessibility of the nearest town. The inset shows the regression of accessibility on population (*10*), which extends Bettencourt's theory to embrace spatial economies of scale based on interactions between cities.

the theory, but it produces more detail pertaining to ways in which cities might depart from the generic scaling derived here.

What is particularly elegant about this theory is its quest for explaining scaling in terms of both the fractal and Euclidean geometry of the city, because it is in terms of these geometries that urban planning has its primary impact. It might be argued that to solve the problems of the city, we need something more than intervention in its physical form (for example, changes in local taxation), but physical plans still mark the way in which we might influence the shape and quality of life in our cities in the least intrusive way.

Bettencourt identifies some revealing tensions in cities that suggest how the physical forms that we might influence can be made clearer, particularly with respect to mobility, transport infrastructure, and density. His theory deals with phenomena within cities, albeit with respect to different sizes of free-standing cities. However, it says little so far about the "system of cities" (*6*) that dominates the modern era of globalization. Given that all humans are predicted to be living in cities by the end of this century (*7*), and that it will often be hard to know where one city ends and another begins (as is true already in many parts of the world), it remains to be shown how well this theory of the individual city will hold up.

In fact, all the rudiments for extending these ideas to systems of cities are in place here. Extension to other cities can be inserted into Bettencourt's theory as some measure of flows into and out of one particular city from and/or to all other cities. This flow might be represented as a gravitational potential or accessibility, with the relative nearness of all places of a particular size moderated by the distance or cost of getting to those places (*8*). It is often hard to define unambiguous boundaries around cities—for instance, in the United Kingdom—but if we use the augmented scaling relation (*9*) and regress this against population for all 283 towns and cities with populations over 10,000 in England and Wales, then we generate superlinear scaling for the system of cities (see the figure) (*10*). Many questions result from such extensions, particularly regarding the robustness of the superlinearity that pertains to the scaling.

In the current thinking about cities, the situation is not very different from that implied by Maxwell in the middle of the 19th century when he said that "the dimmed outlines of phenomenal things all merge into one another unless we put on the focussing glass of theory" (*11*). Bettencourt provides us with the essence of a theory that begins to make sense of how cities change as they scale in a world that is rapidly becoming entirely urbanized.

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