not a terribly good copy I am afraid but it will suffice ...

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CITIES AS SYSTEMS WITHIN SYSTEMS OF CITIES

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THIS PAPER EXAMINES some of the ways by which understanding of cities and sets of cities has been advanced during the first decade of regional science. Originally, I was asked to prepare a paper which reviews the entire range of urban models but, for several reasons, decided to take a more limited view. The Social Science Research Council's Committee on Urbanization has recently completed a comprehensive review of urban studies, to be published shortly as *The Study of Urbanization*,[1] and to attempt to duplicate this work in a short paper would be as foolhardy as the result would be superficial. Other papers to be presented at these meetings will deal with certain kinds of urban models (for example, those related to metropolitan transportation studies, or those involving study of the urban economic base via input-output matrices) and I will not attempt to duplicate what they have to say.

What, then, is the scope of this paper? Three channels which lead towards development of sound urban models are explored and relevant implications drawn. By models, we mean *symbolic* models not those of the *iconic* or *analogue* kinds.[2] Further, the symbolic models of interest are those which provide idealized representations of properly formulated and verified scientific theories relating to cities and sets of cities perceived as spatial systems. Any scientific theory logically comprises two parts: (a) simple inductive generalizations drawn from observable facts about the world, and (b) abstract logical constructs. It is the coincidence of deductions drawn from the logical constructs and inductive generalizations drawn from fact that makes for a valid scientific theory. Ten years ago, urban studies were in an either/or situation either inductive generalizations or logical constructs existed, the former as likely as not produced by urban geographers and the latter by urban economists. As the word "model" became fashionable, both called their products models, but neither had models of theories in the strict sense.

The importance of the last decade has been that the twain *have* met through the medium of regional science. Moreover, the meeting came just when quantitative methods of analysis, facilitated by rapid developments in computer technology, began a technological revolution which has wrought havoc throughout the sciences. What more shattering change could there be than one which facilitates the large-scale studies that lead to specification of strength of belief in inductive generalizations, allow objective testing of the degree of coincidence between inductive generalizations and deductions from logical constructs, and ease replication? The technological advance has meant more, however--virtual elimination of the once lengthy gap between problem

[1] This volume (33) includes review papers by historians, geographers, political scientists, sociologists, economists, and the like.

[2] Ackoff (1) elaborates these terms.

formulation and evaluation of results, sharpening of the questions asked, initiation and completion of experiments of a size unthinkable under earlier technical conditions, and many more.

The meeting, then, was timely. Inductive generalizations could be eased toward theory, logical constructs could be faced with the ultimate test of reality, and new kinds of empiricism and experimentation could be developed. These are the three channels discussed in the following sections of the paper. Examples are presented in an expository rather than a rigorous manner, since each has been elaborated elsewhere. The conclusions of the paper are that urban models are the same kinds of models as appear in other kinds of systems inquiry. Urban theory therefore may be viewed as one aspect of general systems theory. Viable avenues for future urban research might therefore be identified by looking at those other aspects of general systems theory that are relatively well advanced to see how they reached this more developed position.

I. INDUCTIVE GENERALIZATIONS IN SEARCH OF A THEORY

Two of the better-known generalizations concerning cities are the rank-size relationship for sets of cities, and the inverse-distance relationship for population densities within cities. Both had been observed many times when they were formalized as empirical "rules" a decade or so ago, the former as the *rank-size rule* by G. K. Zipf and the latter as the *negative exponential density distance relationship* by Colin Clark. Yet as Isard noted in 1956, "How much validity and universality should be attributed to the rank-size rule is, at this stage, a matter of individual opinion and judgment."[3] Further, although Clark argued that the negative exponential "appears to be true for all times and places studied" he provided no theoretical rationale for his observations, only specified that they might have something to do with transport costs.[4] During the past decade, both inductive generalizations have been brought closer to the status of scientific models, with the range of their validity carefully specified.

Distribution of City Sizes

The rank size rule[5] says that for a group of cities, usually the cities exceeding some size in a particular country

$$p_r = p_1/r^q \tag{1}$$

where p_1 is the population of the largest or first-ranking city, p_r is the population of the city of rank r, and q is a constant.[6] Whence it follows that

$$\log pr = \log p_1 - q \log r \tag{2}$$

so that a plot of rank against size on doubly logarithmic paper should give a straight line with a slope of $-q$.

[3] Isard (34) in connection with a discussion of empirical regularities.

[4] See Berry (12) for review comments.

[5] Berry (8) lists the relevant literature in some detail. Subsequent contributions include those of Bell (5), Friedmann (29), and Ward (51).

[6] If the entire population were urban, then $p_t = p_1 \sum r^{-q}$. See Weiss (52).

Another way of expressing the foregoing is that the frequency distribution of cities by size seems to be highly skewed in the shape of a reversed-*J*. A whole series of probability distributions, among them the lognormal and the Yule, have a similar reversed-*J* shape, each bearing a general family resemblance through their skewness. Each is, in fact, the steady state distribution of a similar simple stochastic process. Could it be that rank-size regularities of city sizes also result from such a stochastic process? The tenor of arguments provided in the past decade is that stochastic processes do indeed provide such a framework, and both the Yule distribution and the lognormal have been proposed of the basis of rank-size regularities.[7] The two are, in fact, so similar that each could obtain when the cumulative distribution of cities, by size, plots as a straight line on lognormal probability paper. Whichever is applicable to the particular case depends upon whether a closed or an expanding system of cities is being considered.

Consider the transition matrix of a stochastic process in which the rows and columns are specified by city-size groups. If the probability density function of each size-class of cities is approximately the same,[8] then the steady-state of the stochastic process will be lognormal if the set of cities existing at the beginning of the process is the same as the set achieving the steady-state at the end. If, however, the smallest size class is augmented by new cities at a fairly steady rate throughout the process, the steady-state is that of the Yule distribution. If growth of cities within the set can be said to occur in small independent increments, with possibilities of growth the same for each size class (growth is the result of "many factors operating in many ways" and occurs such that if city sizes for time period *one* are plotted against sizes for time period n the resulting scatter of points is homoscedastic with a slope of +1), then the basic conditions of such a stochastic process can be said to have been met. One or another constraint leads to the lognormal or the Yule, in the former case a closed system of cities must exist, whereas in the latter the system must go on growing at a steady rate by addition of cities at the lowest level.

A recent study shows that the rank-size regularity applies throughout the world for countries which are highly developed with high degrees of urbanization, for large countries, and for countries such as India and China which, in addition to being large, also have long urban traditions. Conversely, "primate cities" or some stated degree of primacy obtains if a country is very small, or has a "dual economy."[9]

Moreover, additional studies have recently shown that many distributions with some degree of primacy take on more of a rank-size form as level of development and degree of urbanization increase.[10] By virtue of size and complexity, then, countries with rank-size distributions appear to satisfy the condition of "many factors operating in many ways," and increasing complexity ;

[7] Simon (48), Berry and Garrison (7), Thomas (49), Dacey (25), and Ward (51).

[8] That is, so that the "law of proportionate effect" holds.

[9] Berry (8).

[10] Bell (5) and Friedmann (29).

of a space economy certainly brings the city size distribution closer to rank-size. A rank-size regularity is not found when few factors Mold the urban system in a few simple ways: in small countries, where economies of scale accrue in a single "primate city"; or in "dual economies" where one or a few exogenous, colonial, cities of great size are superimposed upon an indigenous urban system of smaller places, etc. In such cases, growth patterns cannot be summarized in the form of a stochastic process of the simple kind outlined above.[11] For all large, complex systems of cities which exist in the world, however, aggregate growth patterns do conform to such a stochastic process, so that one macroscopic feature of these systems is a rank-size regularity of city sizes. The regularity may, in turn, be "explained"[12] by the stochastic process.

Urban Population Densities

No city has yet been studied for which a statistically significant fit of the expression

$$d_x = d_0 e^{-bx} \tag{3}$$

does not obtain. In this equation, which was derived empirically by Colin Clark, d_x is population density d at distance x from the city center; d_0 is central density, as extrapolated into the city's central business district; and b is the density gradient.

$$\ln d_x = \ln d_0 - bx\,. \tag{4}$$

Alonso and Muth have provided a satisfactory "explanation"[14] of the observed regularity recently in terms of the rent-transport-cost-trade-off of individuals in different stages of the family cycle at different income levels and at different distances from the city center.[15] Thus, what Clark speculated might have something to do with transport costs, when he advanced the regularity a decade ago, in fact, does so. Apparently, the bid-rent function is steeper for the poorer of any pair of households with identical tastes in the American city, so the poor live toward the city center on expensive land consuming little of it and the rich on the periphery consuming much.[16] The negative exponential shape of the decline stems from the nature of the production function for housing and the shape of the price-distance function.[17] Expression (3) is thus an equation of some generality which can be derived as a logical implication of the theory of the urban land market.

This being so, a variety of conclusions may be drawn. For example, the population residing at distance m from the city center is

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[11] Unless the process works, for example, to a random power of size, as with the log-lognormal, see Thomas (49).

[12] Nagel (42) discuss the various modes of scientific explanation and the role of explanation in science.

[13] Berry (12) lists the relevant literature. Also see Winsborough (54).

[14] Alonso (2) and Muth (41). Also see 12 above.

[15] *Ibid.*

[16] Alonso, *op. cit.*

[17] Muth, *op. cit.*

$$p_m = \int_0^m d_0 e^{-bx} (\pi 2x) dx \tag{5}$$

which becomes

$$p_m = 2 d_0 \pi b^{-2} [1 - e^{-bm}(1 + bm)] . \tag{6}$$

This implies that the population pattern of an urban area can be described by two parameters alone, b and d_0. Winsborough has called the former a measure of the *concentration* of the city's population and the latter an index of its *congestion*.[18]

Now for any set of cities and for any particular city through time, another empirical expression holds[19]

$$b = ap^{-c} . \tag{7}$$

Thus, b is in turn a function of city size. Central density, d_0, is on the other hand apparently a function of the form of the city as established at the particular stage at which it grew and is thus directly related to the city's age.[20] Knowing the population of a city and its age, it is possible to predict fairly closely the pattern of population densities within it.

In any system of cities for which the rank-size regularity obtains, the population p of a city of rank r, p_r, is a function of only p_1 and q, equation (1). Hence, b must likewise be a function of p_1 and q, equations (1) and (7). The distribution of population within cities is a function of the position of these cities within the entire system of cities and of age. If the larger system is Yule in form, age is simply the generation of the underlying stochastic process at which the city entered the system, so that congestion, d_0, as well as concentration, b, is given within the framework of the larger system. The preceding statement can thus be modified to read: the distribution of population within cities is a function of the position of these cities within the entire system of cities at some point in time, and the period of time for which they have been within the system.

II. LOGICAL CONSTRUCTS IN SEARCH OF A TEST

The preceding two models account for the size and the distributional characteristics of urban populations, but they say nothing of the locations of the cities concerned. Three sets of reasons for cities have been advanced, each with locational parameters more or less explicit: cities as strategic locations on transport routes; cities as the outcome of local concentrations of specialized economic activities; and cities as "central places" performing retail and service functions for surrounding areas. Only the latter is of interest here.

Central Place Theory

Central place theory[21] was formulated by Walter Christaller as a "general

[18] Winsborough (54).

[19] Berry (12), Weiss (52), and Newling (43).

[20] Winsborough, *op. cit.*

[21] Berry and Pred (9). Later studies include (10), (11), (13), (22). See also the parallel speculations of Rashevsky (46), (47).

purely deductive theory" designed "to explain the size, number and distribution of towns" for reasons which also made it "the theory of urban trades and institutions."[22] A decade ago this theory was perhaps the only one concerning systems of cities that was at all well developed.[23] At that time, although many empirical studies of central places had been completed, the fact that no satisfactory test of the theory had been made largely reflects the fact that investigators looked for examples of theoretical implications drawn simply for exemplification by Christaller under the assumption of an isotropic plane. There also was a lively debate as to whether certain of the most fundamental theoretical implications, for example that of a hierarchy of central places, had any empirical validity. It has only been during the last decade that such questions have been settled. A thorough review of most spects of the topic is to be found in *Central Place Studies. A Bibliography of Theory of Applications*, the first of the Regional Science Research Institute's Bibliography Series, and so will not be repeated here.[24] Subsequent to the *Bibliography*, the various postulates of the theory were drawn together in a model. Since the model appears to have some generality (implications drawn from the model have been verified independently, for example) it will be presented here.[25]

The model applies to systems of central places in which the elements are viewed aggregatively. A set of inequalities supplements the model, however, and these empirically derived expressions link aggregate patterns to local arrangements of central places under specified conditions of population density by specifying expectations as to the steps of the central place hierarchy. Random variations from ideal steplike patterns of central places in a series of local areas, combined with logical changes in location of the steps according to population density, interact to produce the regularities which may be observed in the aggregate. The definitions, equalities, structural equations, and implications of the model follow without lengthy comment.

Definitions:

p_t = the total population served by a central place;

p_c = population of the central place;

p_r = rural population and population of lower level centers served by the central place,

A = area of the trade area served;

Q_t = population density of the area served;

Q_r = population density of those parts of the area served lying outside the central place;

T = number of central functions performed by the center, and since central functions enter in a regular progression and can be ranked from $1 \ldots T$ in decreasing order of ubiquity, also the highest level central function performed by the center;

E = number of establishments providing the T types of business;

[22] Christaller (21).

[23] Berry and Pred *op. cit.*

[24] *Ibid.*

[25] See (10) and (14).

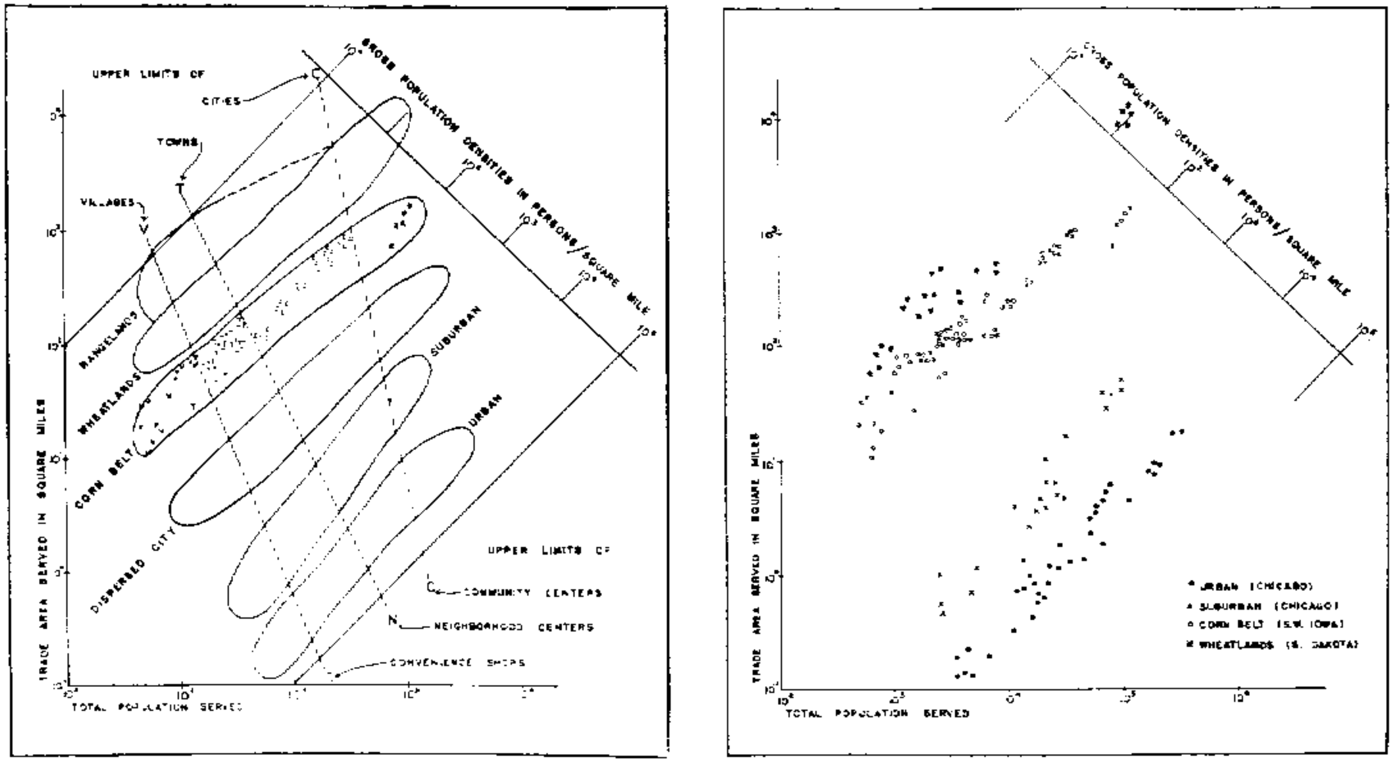

FIGURE 1. GROSS TRADE AREA CHARACTERISTICS IN THE UNITED STATES.

The right hand figure shows sample data for central places in Chicago, Chicago's suburbs, the corn belt of southwestern Iowa, and the wheatlands of South Dakota. The "sausages," in the left-hand illustration, encircle the scatters of points in each case, adding data for the "dispersed city" of southern Illinois, and the rangelands around the Black Hills of South Dakota. Note that, in the corn belt case, the factor analytic classification of the central places has been added, to illustrate how the inequalities representing the upper size limits of centers of each class under varying conditions of population density actually operate.

D_m = maximum distance consumers will travel to a central place of size T, *or* the range of good T.

Equalities:

$$p_t = p_c + p_r \qquad \text{(E1)}$$

$$p_t = AQ_t \qquad \text{(E2)}$$

$$p_r = AQ_r \qquad \text{(E3)}$$

$$A = kD_m^q \qquad \text{(E4)}$$

Figure 1 shows, in five distinct study area in the United States,[26] equality (E2). In each case, total population and total area served slope upwards to the right on doubly logarithmic paper with a slope of +1. Differences between study areas are simply a function of population densities.

Structural Equations:[27]

$$\log p_c = a_1 + b_1 T \qquad (1)$$

$$\log D_m = a_2 + b_2 T \qquad (2)$$

$$\log E = a_3 + b_3 \log p_t \qquad (3)$$

These structural equations hold in any study area (i.e. at any level of density) and relate the population of a market center to the variety of central functions performed for surrounding areas, the drawing power of the center to its offerings, and the number of separate establishments performing the T functions (E exceeds T for all except the smallest villages and hamlets) to the total population served to account for nonbasic demands for goods and services from the population p_c as well as basic demands generated by the population of the area served, p_r.

Implications:

$$p_c = p_t^s w^{-s} Q_t^{-s} \qquad (4)$$

where

$$w = k[\log^{-1}(q b_1^{-1}(a_2 - a_1 b_2))]$$

and

$$s = (b_1)/(q b_2)$$

As total population served increases, the central population comes to assume an increasing proportion of the total, but this tendency varies inversely with population densities.

$$A = w p_c^x \qquad (5)$$

where

$$x = s^{-1}$$

Area served increases exponentially with size of center.

$$E = m Q_t^{b_3} p_c^{b_3} \qquad (6)$$

where

$$\log m = a_3 + b_3 \log w$$

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[26] See (10) or (11) for details.

[27] Only a sample of the structural equations necessary to facilitate the present discussion is given here.

Total number of establishments varies exponentially with both size of center and total population densities.

These and similar structural equations and implications have now been verified in several studies[28] and appear to be reasonable summary of many of the aggregate features of central place systems. Each has particular implications within the framework of central place theory as well, particularly as it has been generalized. However, a set of inequalities is needed in combination with Figure 1 to lay out the steps of the central place hierarchy as it is found in local areas at different levels of density. These inequalities were established empirically by factor analyses of the functional structure of central places in each of several study areas, to determine the hierarchy individually within each of those areas, and then by discovering, unexpectedly, that limits to each of the levels varied consistently across the set of study areas as population density varied. With the second subscripts v referring to villages, t to towns and c to cities, these inequalities are:

$$\log A_{tv} \leq 10.4 - 2.67 \log p_t \tag{I1}$$

$$\log A_{tt} \leq 9.3 \quad 2.067 \log p_t \tag{I2}$$

$$\log A_{tc} \leq 22.25 - 4.75 \log p_t \tag{I3}$$

They are inserted into Figure 1, and, in the case of the corn belt study area, the individual observations are identified as they were classified in the factor analysis.[29]

III. INNOVATION UNDER TECHNICAL IMPETUS

The decade has seen a variety of innovations, most facilitated by rapid developments in computer technology that made feasible kinds of research that could never have been contemplated prior to the developments. Beginnings are to be seen in the construction of urban simulators that will facilitate study of cities and sets of cities in laboratory-type experimental situations.[30] The most successful attempts so far have been those of Chapin in studies of land development[31] and Morrill in analyses of changing central place patterns,[32] although this statement is not intended to denigrate attempts along these lines in current urban transportation and economic studies. Out of these studies, particularly those undertaken in Chicago, Pittsburgh, and the Penn-Jersey region, and also those of The RAND Corporation and Resources for the Future, Inc. will surely emerge models of some predictive power and experimental capability. Another paper at these meetings considers this topic however, recognizing what may be the most significant new dimension to urban research added during the past decade. We will concentrate here on another topic, the new empiricism of the decade, stimulated by advancing computer technology

[28] See (10) and subsequent studies as yet unpublished by Karaska, Pitts, Murdie, and others.

[29] The factor analytic results are presented in (14) and (10).

[30] Garrison (30) has one of the first presentations.

[31] See (20).

[32] See (38) and (39).

and consequent diffusion of multivariate analysis throughout the social sciences. We focus here on one form of multivariate analysis, *factor analysis*, and briefly review how, in the form of social area analysis, it has facilitated studies of the internal structure of cities.

Social Area Analysis

Social area analysis[33] is one approach to the classic problem of urban ecology, the succinct description of the location of residential areas by type within cities, in terms meaningful to persons interested in social differentiation and stratification. Over the years, several constructs have been developed in this context:[34] Hurd's concept of urban growth proceeding according to two patterns, central growth and axial growth; Burgess' concentric zone hypothesis of the location of residential areas by type, stemming from the nature of a growth process that proceeds outwards from the city center, accompanied by waves of residential invasion and succession; Hoyt's emphasis upon the axial growth of higher-income neighborhoods outwards from the city center along some sector; and Harris' and Ullman's notions of the multiple nucleation of the city. Both the social area analysts and their critics[35] have emphasized the difficulty of testing these hypotheses with the wide variety of socio-economic data available, for example, from censuses. Which variables should be used in the test? Will the story told by different but presumedly related variables be the same? What in fact are the stories told about the structure and differentiations of urban neighborhoods by the wide range of census data available?

Factor analysis can provide answers to questions of the latter kinds. Let us review the basic features of the method. Consider a data matrix ${}_nX_m$ in which are recorded the data for n observations (say, census tracts) over m variables. If the column vectors of X are normalized and standardized to yield ${}_nZ_m$ then $Z^TZ={}_mR_m$ which is, of course, the correlation matrix of the m variables. Since the column vectors of X were standardized, R is the variance-covariance matrix of Z, and the trace of R, equaling m, is the total variance of the m variables.

Now assume that each of the m variables is regressed in turn upon the $m-1$ remaining. For each is then available a coefficient of determination expressing how much of its variance is held in common with the $m-1$ other variables; in factor analysis these coefficients of determination are called communalities, and denoted h^2. For each variable, then, in its standardized form $1.0-h^2=u^2$ is the proportion of variance unique to the variable. A diagonal matrix U^2 can thus be formed with individual u^2s along the diagonal. It follows that $[R-U^2]$ has communalities on its diagonal, and the trace of $[R-U^2]$ is the total common variance of the m variables. This total common variance plus the trace of U^2 equals m, the total variance.

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33 Bell (6) has an excellent review.

34 See the review by Anderson (3).

35 The Duncans write "students of urban structure have lived for some time with the uncomfortable realization that their theories, or rather their abstract, schematic descriptions, of urban growth and form are not very susceptible to empirical testing" (26).

Principal axes factor analysis provides a procedure whereby a matrix ${}_mA_r$ may be found such that

$$[R - U^2] = AA^T \quad (1)$$

$$A^T A = \Lambda \quad (2)$$

The dot product of each row vector of A yields one of the communalities, and the inner product of any pair of row vectors reproduces a correlation. Λ is a diagonal matrix, which implies that inner products of pairs of column vectors of A are zero. Such vectors are thus orthogonal (uncorrelated). The dot product of each column vector yields an eigenvalue λ. Since the sum of the eigenvalues must equal the sum of the communalities, these eigenvalues represent another way of parceling up the total common variance, the one (communalities) relating to the amount of the total common variance contributed by the association of any one variable with all other variables, the other (eigenvalues) to that part of the total attributable to one of the column vectors of A. These independent column vectors are the factors of factor analysis, the principal dimensions of variation underlying the original body of variables m.

Individual elements of A are factor loadings, the correlation coefficients between the original variables and each of the underlying common dimensions. The property of orthogonality of the dimensions is useful, because it means that each of the dimensions accounts for a different slice of the common variance, which slices are additive in any reconstruction of the whole; such additivity was not a property of the original intercorrelated m variables. Each dimension summarizes, then, one pattern of variation (one story told by) the original m variables. A further step which is useful is to form

$${}_nS_r = ZA\Lambda^{-1} \quad (3)$$

In S, the individual s_{ij} are factor scores of the original observations on each of the new dimensions formed by the analysis. S expresses all associations and common patterns found in X, but in a simpler form.

Factor analysis of census data for a whole series of United States cities by social area analysts has led to the conclusion that three dimensions are all that is required to summarize the stories told by the characteristics recorded for census tracts by the census. Study of the correlations between the original variables and the three dimensions has also revealed remarkably stable patterns from one city to another. One factor was consistently highly correlated with income, education, occupation, and wealth. A second was related to family structure, fertility, type of household, and position of women in the labor force. Finally, a third was associated with ethnic and racial structure of the population, age and sex composition, and measures of deterioration and blight. Speculation about the meaning of these regularities led social area analysts to identify the first as depicting variations in the *social rank* of individuals and families, the second as representing variations in the *urbanization* or *family status* of neighborhoods, and the third as resulting from *segregation*. The factor scores of tracts on these three dimensions could be used to characterize neighborhoods, since the three dimensions appear to be those responsible for the basic features of urban differentiation and stratification.

If the latter statement is true, then the three dimensions should enable research workers to test some of the classical constructs concerning such urban differentiation and stratification. A first study along these lines has revealed that factor scores of tracts with respect to social rank are differentiated in a sectoral fashion, as they should be if Hoyt's concepts apply, and that factor scores on urbanization and family status are differentiated in a concentric fashion, as they should be if Burgess's ideas are valid.[36] However, spatial variations in segregation show no regularity but are specific to each case. As Hurd had speculated much earlier, then, concentric and axial patterns are additive and independent sources of urban differentiation from city to city, with spatial variations specific to each city added by the third dimension of segregation.

It is clear that although social area analysts began simply in a "look-see" manner, with later work facilitated by advancing computer technology, their work has now laid the bases for a spatial model of the internal socio-economic pattern of cities in which the relevance and role of the traditional concepts are clear.[37,38]

IV. A SYSTEMS FRAMEWORK

The previous findings point in one direction: that cities and sets of cities are *systems* susceptible of the same kinds of analysis as other systems and characterized by the same generalizations, constructs, and models. *General systems theory* provides a framework for such inquiry into the nature of systems; indeed Boulding calls it the "skeleton of science." Further, *information theory* has come to the fore as one of the foundations of general systems theory, contributing the two complementary ideas of *entropy* and *information* to the vocabulary of general systems research.[39] Entropy is achieved in the steady state of a stochastic process and is at its maximum if this process is unconstrained. Information is a measure of the order present if some systematic pressures for organization constrain the operation of the stochastic process.

Curry[40] has shown that given Z settlements, with Z_i having a population i, the numbers of ways people can be distributed among settlements is

$$p = Z! / \prod_{i=0}^{n} Z! \qquad (0 \leq i \leq n) \tag{1}$$

and in a large system the entropy E is given by

[36] Anderson, *op. cit.*

[37] This, in spite of criticism (26), has been the *accumulative* result.

[38] It is worthwhile to note some of the other contributions made possible by factor analysis: (a) more general urban typologies (40); (b) clearcut evidence of the hierarchy of central places as an additive class system (10), (14); (c) multivariate regionization (31); and (d) metropolitan structure (32).

[39] Bertalanffy (16), (17); Boulding (19); and Beer (4).

[40] Curry (23). Other cases he examines are the spacing of nearest neighbors, also see Dacey (24), the spacing of nearest neighbors of the same size, and the per cent of manufacturing in an urban labor force.

$$E \underset{\text{def.}}{=} \log p = Z \log Z - \Sigma Z_i \log Z_i \qquad (2)$$

E is maximized when

$$Z_i = (Z/N)e^{-(i/N)} \qquad (3)$$

in which equation N is the mean population per settlement, or $N = n/Z$. Now if S is the size of the largest city,

$$Z_{i<S} = S(1 - e^{-(i/N)}) \qquad (4)$$

in which case

$$E_{\max.} = Z \log (eN) \qquad (5)$$

and the most probable state of the system, giving maximum entropy, is one in which, given the size of the largest city, the probability of the $(q+1)$st city having a population which is a given ratio of the qth city is a constant. Under these conditions, the sum of the logarithms is a maximum, and of course it is the condition satisfied when the rank-size rule for cities obtains. If a system of cities assumes rank-size, then, entropy has been maximized and it has assumed its most probable steady-state.[41]

On the other hand, organization exists due to pressures for order in central place systems. If the per cent change in establishments in central places is constant with each addition of new business types, then[42]

$$dE/EdT = k\,. \qquad (6)$$

Integrating yields

$$\log E = k_1 T + c_1\,. \qquad (7)$$

If similar percentage ratios exist for the sizes of central places p_c then

$$\log p_c = k_2 T + c_2\,. \qquad (8)$$

From the above

$$T = K_1 \log E - C_1\,. \qquad (9)$$

$$T = K_2 \log P_c - C_2\,. \qquad (10)$$

Now the equation

$$I = K \log (\text{number of states}) \qquad (11)$$

has been identified as one measure of macroscopic negentropy, the inverse of entropy. It follows that the number of business types T is an index of the amount of information present in a set of establishments located in central places or of the population of those places. This is consistent with the use of types of functions to identity and classify the central place hierarchy. Many attempts have been made to assess the "centrality" of central places. It would seem that number of types of business, information content, provides such an index. In southwestern Iowa, very strong fits to (9) and (10) are found.

$$T = 55.56 \log E - 58 \qquad (r^2 = 0.96)\,. \qquad (12)$$

$$T = 50.00 \log p_c - 105 \qquad (r^2 = 0.91)\,. \qquad (13)$$

[41] Curry points out that entropy in the same system constrained such that persons had to be allocated in threes, as families, would be $H' = Z \log (eN/3)$. Hence, a measure of order is $R = 1 - H'/E_{\max}$.

[42] Odum (44), Berry (11).

indicating that where urban centers are almost exclusively central places, necessary empirical bases for these arguments are to be found. It will be readily apparent that the above equations are compatible with those presented earlier for central place systems in Section II. Lösch and Christaller postulate such constant percentage relationships also with the addition of *levels* to the regular hierarchy ($k-3$, $k-4$, $k-7$ networks and their implications); related measures of information should therefore exist for the order maintained by the step-like nature of the hierarchy.

It is not difficult to extend similar arguments to the situation within cities.[43] For example, urban population densities settle down to a most probable state in which densities are ranked with distance from the city center. Conversely, the model of central place systems also applies, indicating that certain aspects of urban life are constrained from reaching their most probable state.

Maruyama[44] has speculated about an apparent contradiction of the second law of thermodynamics in social phenomena, including those of cities. According to the second law, an isolated system will most probably trend to its most probable state, even if it begins in an inhomogeneous state. He points out that cybernetics, the study of equilibrating systems, considers many cases of self-regulation such that deviations are counteracted and the system is brought back toward its equilibrium, usually a most probable state under constraint. But many instances can be cited in which feedback does *not* lead to self-correction towards some preset equilibrium (morphostasis). Rather, progressively greater contrasts appear, as between Myrdal's "rich lands and poor," or with progressively greater centralization of urban functions in fewer larger cities, or when the "growth of a city increases the internal structuredness of the city itself," in Maruyama's words. These are all examples of deviation *amplifying* processes (morphogenesis) which run counter to the second law.

Whether or not a system trends toward maximum entropy because processes working are deviation correcting, or toward maximum information because the processes are deviation, and therefore structure-amplifying, apparently depends upon the nature of the causal relationships at work and of their feedback characteristics. Maruyama concludes that any system, together with the subsystems into which it may be partitioned, contains many examples of both deviation-correcting and deviation-amplifying processes. One subsystem may be becoming more highly organized, another may be approximating its most probable state. To understand the system as a whole demands that each of the subsystems be understood, as well as the relationships between them.[45]

So be it in the urban field. It is clear that cities may be considered as systems- entities comprising interacting, interdependent parts. They may be studied at a variety of levels, structural, functional, and dynamic, and they may be partitioned into a variety of subsystems. The most immediate part

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[43] Meier (37).

[44] See (35) for a review statement of Maruyama's ideas and other references of interest.

[45] Maruyama provides an example of the operation of deviation-amplifying mutual causal processes in a two-dimensional spatial distribution, and his discussion of systems, subsystems, and feedback is phrased in terms of cities.

of the environment of any city is other cities, and sets of cities also constitute systems to which all the preceding statements apply. For systems of cities, the most immediate environment is the socio-economy of which they are a part.

Whereas progress has been made in understanding various facets of these systems and subsystems, for other facets we stand much as we did a decade ago. In a systems framework, we should no longer worry about apparent contradictions between the kinds of conclusions reached for different subsystems, i.e. between the distribution of city sizes and the functional arrangement of market centers in a hierarchy, however, for the difference is understood to be one of the relative balance of entropy-approximating or order-generating processes in various parts of the system. In contradistinction, however, we have very little understanding of how to put these different patterns together in more general models that are broad in scope. Sound models are providing the building blocks, but maximum progress during the next decade awaits the architectural systematizer.

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