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The city size distribution debate: Resolution for US urban regions and megalopolitan areas

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ABSTRACT

Four phases of interest in the distribution of city sizes are identified and current conflict in the literature is shown to be a consequence of poorly-selected units of observation. When urban regions are properly defined, US urban growth obeys Gibrat's Law and the city size distribution is strictly Zipfian rank-size with coefficient $q = 1.0$. Care has to be taken with definition of the largest urban-economic regions, however; the fit in the upper tail of the distribution is best when they are recognized to be megalopolitan in scale.

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1. Introduction

Following the proclamation of a "New Economic Geography" by Krugman (1991a,b) there has been a renewed interest in city size distributions, led by vigorous debates among economists and between them and others, especially physicists. Researchers have asked about the theoretical underpinnings of Zipf's rank-size rule (Córdoba, 2008a; Duraton, 2006, 2007; Gabaix, 1999; Rossi-Hansberg and Wright, 2007); whether urban growth obeys Gibrat's Law of Proportionate Effect (Córdoba, 2008b; Cuberes, 2009, 2011; Eeckhout, 2004, 2009; González-Val et al., 2010; Ioannides and Overman, 2004; Ioannides and Skouras, 2009; Levy, 2009); whether Zipf's rule is best captured by a Pareto or a lognormal distribution, and the proper testing procedure for distinguishing them (Bee et al., 2011; Garmestan and Allen, 2008; Terra, 2009; Giesen et al., 2010; Malevergne et al., 2009, 2011; Nitsch, 2005); the proper units of observation (Cladera and Arellano Ramos, 2011; Eeckhout, 2004; Holmes and Lee, 2009; Jiang and Jia, 2010; Nitsch, 2005; Rozenfeld et al., 2010; Ye, 2006); and whether these units of observation are independent or spatially autocorrelated (Favaro and Pumain, 2011; Ioannides and Overman, 2004). We place these debates in their historical settings and then, using the Economic Areas (EAs) defined by the Bureau of Economic Analysis of the US Department of Commerce as units of observation, demonstrate that US urban-regional growth conforms to Gibrat's Law and that the size distribution is Pareto in the upper tail, meaning that Zipf's Law obtains. However, we find that this solution understates the size of the nation's five largest urban regions, which leads us to conclude that they must be megalopolitan in scale (Gottmann, 1961). By grouping EAs to approximate such areas as Gottmann's "Boswash" and "Sansan" we improve the fit of the model in the upper tail, which reinforces our conclusion that once the observational units are properly defined the US city size distribution in Zipfian in the strict sense.

2. Four research epochs

The distribution of city sizes has been a topic of interest for at least the last century. Four phases of interest can be identified, including the surge of activity by the New Economic Geographers in the last two decades.

2.1. An interesting empirical regularity

Initial interest was sparked by Felix Auerbach's finding that for the United States and five European countries, city population conformed to the relationship $p_iRi = A$, where A is a constant, p_i is the population of the cities in size-class i, and R_i is the rank of class i when the size-classes are ordered from 1 to n by population size (Auerbach, 1913). Later, Lotka (1925) found that a better fit for the 100 largest cities in the US in 1920 was provided by $p_r r^{0.93} = 5{,}000{,}000$ where p_r is the population of the rth ranking city and cities are ranked from 1 to n in decreasing order of size. Singer

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(1936) rewrote this relationship as $rp_r^{\alpha} = A$. The coefficient α is obtained by fitting the equation $\log r = \log A - \alpha \log p_r$ to the data. This can written as $r = Ap_r^{-\alpha}$, where rank r is also the number of cities with population p_r or greater.[1] This last expression is the survival function of a Pareto distribution with exponent α, cumulated from large to small.

2.2. Zipf's Law

The second phase of interest began with Zipf's restatement of the relationship between population and rank as $p_r = Kr^{-q}$, the rank-size distribution, going beyond Jefferson's (1939) proposed "law of the primate city" (Stewart, 1947; Zipf, 1941, 1949). In the special case where the exponent $q = 1$, this is known as Zipf's Law; $p_r = K/r = p_1/r$. Most researchers during this second phase estimated the Zipf model ($\log p_r = \log K - q \log r$) rather than the Pareto-form equation,[2] but the relationship between the two is straightforward, and simple algebraic manipulation shows that $q = 1/\alpha$. As $\alpha \to inf$, $q \to 0$, and all cities are equal size. If $\alpha = q = 1.0$, Zipf's Law holds strictly.

Zipf and Stewart precipitated a rush of subsequent literature (e.g. Berry, 1961, 1964, 1971) that concluded with an ambitious cross-national analysis undertaken by Rosen and Resnick (1980), a comprehensive literature review by Carroll (1982), and several papers by Alperovich (1984, 1988, 1989). Rosen and Resnick estimated Pareto coefficients for 44 countries c.1970.[3] Three quarters of the cases had α coefficients greater that unity and therefore had urban populations more evenly distribution than predicted by Zipf's Law. This finding was shown to be sensitive to data definitions: Pareto coefficients were closer to unity when the observations conformed more closely to integrated urban-economic regions rather than to legally-defined entities, a point to which we will return later since it has been overlooked by many of the more recent contributors to the field.[4] They concluded that the Pareto distribution was the best general description of rank-size data.

Despite their rich empirical findings, Rosen and Resnick ended their investigation with a plea, however. The empirical work lacked one crucial element, they said: a rigorous theoretical model explaining the size distribution of cities. By "theory", true to their training as economists, they meant an intuitively appealing causal story that stands on an axiomatic foundation. This question of theoretical explanation also was at the heart of Carroll's literature review. After examining the body of empirical work to see whether any clear choices of theoretical perspective were indicated, he concluded that there were not (Carroll, 1982, p. 37). It seems that at this point, he said, we do not need new models but instead some basis upon which to rule out several of the existing ones.

2.3. Building a body of theory

This spirit of skepticism about an acceptable theoretical basis precipitated the third research phase. Krugman, reflecting the economist's preference for axiomatic logic, expressed this feeling in a number of places. "Attempts to match economic theory with data usually face the problem that the theory is excessively neat, that theory gives simple, sharp-edged predictions, whereas the real world throws up complicated and messy outcomes. When it comes to the size distribution of cities, however, the problem we face is that data offer a stunningly neat picture, one that is hard to reproduce in any plausible (or even implausible) theoretical model" (Fujita et al., 2001, p. 215). A new group of scholars, largely economists, took up this theoretical challenge, accepting Zipf's Law as the outcome of growth processes that satisfy Gibrat's Law[5] and seeking to provide an axiomatic economic foundation for the law by providing one for Gibrat.

The first attempt to build such an economic theory was that of Gabaix (1999, see also Gabaix and Ioannides, 2003), who began by confirming the empirical regularity by fitting the Pareto form to the 135 largest US Standard Metropolitan Areas listed in the *Statistical Abstract of the United States* in 1991, yielding: $\ln r = 10.53 - 1.005(\pm 0.010) \ln p_r$. With $\hat{a} = 1.005, 1/\hat{a} = q = .995$. Since $\hat{a}$ is very close to 1 this implies that $Pr(p > p_i) \sim A/pi$, which further implies that distribution of city sizes can be described by a power law, at least in the upper tail. Feeling secure in the empirics, Gabaix then assumed a fixed number of cities that, for a certain range of sizes, grow stochastically, with the process described by a common mean and variance, i.e. the homogenous growth process that is a reflection of Gibrat's Law. In the steady state, this process produces a distribution of city sizes that follows Zipf's Law with a power exponent of 1, thereby "transforming a quite puzzling regularity, Zipf's Law, into a pattern much easier to explain, Gibrat's Law ... models of city growth should deliver Gibrat's Law in the upper tail" (Gabaix, 1999, p. 742).

What type of urban growth process can be described by a common mean and a common variance? Gabaix postulated that growth shocks are *iid* and impact utilities both positively and negatively.[6] Differential population growth was assumed to result from migration which, in equilibrium, forces utility-adjusted wages to equate at the margin. Adding the assumption of constant returns to scale in production technology yielded an expression in which expected urban growth rates are identical across city sizes and variations are random normal deviates, the exponent α tends to 1, and Zipf's Law results.[7]

Gabaix's paper was not without its critics. Physicists Blank and Solomon (2001) asserted that Gabaix's formulation fails to con-

[1] This is of course a member of the class of power laws in which the value of the output y is proportional to some power of the input $x = y = f(x) \sim x^a$. It follows that r/n is the proportion of cities of at least population p_r, and $r/n = (A/n)p_r^{-\alpha}$ which is equivalent to writing the probability that a city's population p exceeds p_i as $Pr(p > pi) = (A/n)pi^{-}\alpha$.

[2] Simon (1955) proposed the Yule (1924) distribution as an alternative when the distribution is that of cities organized by size class. Berry and Garrison (1958) evaluated several models and favored Simon's proposal, as did Krugman (1996, p. 399). For a more recent evaluation see Zanette (2008).

[3] Allen (1954) had undertaken a similar exercise at an earlier date. A recent example is Soo (2005).

[4] One exception in Nitsch (2005), who reviewed 515 estimates of Zipf's Law from 29 previous studies. He writes (p.91): "My results strongly confirm that: for agglomeration data the average Zipf estimate is considerably smaller (and closer to one) than for city data; the difference in means is statistically highly significant." Rozenfeld et al. (2011, p. 2223) add:
"An open question that ... is the following: why is the distribution of "legal" cities broadly lognormal (Eeckhout, 2004; Levy, 2009; Eeckhout, 2009; Ioannides and Skouras, 2009), while the distribution of geography-based "agglomerations" is quasi-Zipf distributed?"

[5] Gibrat's Law states the size S of an observation is independent of its rate of growth $dS(t)/dt$. This "Law of Proportionate Effect" produces an outcome in which the logarithms of S are distributed according to the normal distribution, i.e. a lognormal distribution (Aitchison and Brown, 1957; Gibrat, 1931). Consider the growth equation $p_{it} = \alpha_{it} p_{it-1}$ where p_{it} is the size of city i at time t and α_{it} is a random positive growth rate. Taking the logarithm and iterating produces $\log p_{it} = \log p_{i0} + \alpha_{i1} + \alpha_{i2} + \cdots$. If the α_{it} are independent and identically distributed (*iid*) random variables with mean A and standard deviation B, the central limit theorem gives $\log P_{it} = tA + \sqrt{t} \cdot B \cdot C$, where C is a Gaussian random *it* variable $N(0, 1)$. By assuming that the stochastic process for a given city over multiple time periods is equivalent to a sample of many cities at a point in time, the resulting distribution of city sizes will be lognormal. Because Gibrat's Law models a given city's growth as a random walk, no steady state is possible without a modification, however, in which $p_{it} = \alpha_{it} p_{it-1} + \varepsilon_{it}$ and $\varepsilon_{it} > 0$. The effect of this modification is to transform the lognormal into a Pareto distribution. See Malevergne et al. (2009, 2011).

[6] Other shock-based approaches include Zanette and Manrubia (1997), who generated intermittent spatiotemporal structures from multiplicative and diffusion processes in which citizens of the same city are subject to the same aggregate shocks, and Marsili and Zhang (1998), who centered their shocks on migration processes. In both cases, numerical simulations showed that Zipf's Law emerges as the steady state.

[7] Following on a suggestion by Berry and Garrison (1958), Hsu (2010) begins with central place theory as an equilibrium entry model and identifies the conditions under which a hierarchical city size distribution will follow a power law.

verge on a power law. They identified what they believed to be a missing assumption and then, following Levy and Solomon (1996), demonstrated how a random growth model can generate a power law.[8] In light of this criticism, and because it lacks economic content, other economists have attempted to develop alternative axiomatic models together with alternative formulations of random growth models that satisfy Gibrat's Law. Duranton, for example argued that several economic mechanisms could equally well replicate the observed patterns and that as a consequence the challenge was no longer to focus on the exact shape of the city-size distribution, but instead to evaluate what the real drivers of urban growth and decline are, in particular the churning of industries across cities (Duraton, 2006, 2007). In the same vein, Eeckhout (2004) proposed an equilibrium theory of local externalities in which the driving forces, assumed to be random local productivity processes and perfect mobility of workers, explained lognormality of the city distribution, and in Rossi-Hansberg and Wright's (2007) urban growth model cities arise endogenously out of a tradeoff between agglomeration forces and congestion costs. Urban structure was shown to eliminate local economies of scale while mobility ensured that the marginal product of labor is scale independent, yielding constant returns to scale that, in the aggregate, produced balanced growth and therefore a city-size distribution describable by a power law with an exponent of 1.

Following this, Córdoba (2008a,b) sought to isolate a standard urban model with localization economies that can generate a Pareto city-size distribution. His key statistical result was that the model must have a balanced growth path in which all cities have the same expected growth rate. This is achieved when, under one of three conditions (the elasticity of substitution between goods is 1, externalities are equal across goods, or a knife-edge condition on preference and technologies is satisfied) city growth is independent of size and the steady-state distribution of the fundamentals, preferences, and technologies for different goods is Pareto.[9] The economic explanation is that the standard model can generate a Pareto distribution of city sizes either when preferences for goods follow reflected random walks and the elasticity of substitution between goods is 1, or when total factor productivities in the production of different goods follow reflected random walks and increasing returns are equal across goods. Under such conditions, the steady state distribution of the exogenous driving force must be Pareto. Thus, ''Gibrat's Law is not just an explanation of Zipf's Law ... it is the ... explanation'' (Córdoba, 2008a, p. 178).

2.4. A stochastic steady-state

The case might seem to be closed, but at this juncture the recent literature has taken two critical turns. One group of scholars argues that an economic theory is not required because skewed distribution functions of the city size type are uniquely stochastic steady states. Another group has engaged in a confrontational debate about whether Gibrat's Law describes urban growth and whether the size distribution is better classified as lognormal or Pareto.

In the first group, Axtell and Florida (2001) have produced Zipfian steady states via agent-based modeling.[10] Numerical solutions yield empirically-accurate firm characteristics: a right-skewed distribution of firm sizes, a double-exponential distribution of growth rates and variance in growth rates that decrease with size according to a power law, which in turn yield city-level macro behavior that satisfies Gibrat's Law and produce the Zipf rank-size distribution as a steady state. In other words the result is self-organized complexity characterized by power law frequency-size scaling (Turcotte and Rundle, 2002).

In the same vein, Semboloni (2001) has modeled multi-agent interactions via a probabilistic law to obtain opposing goals that conform to the Zipfian processes of unification and dispersion and used numerical analysis to reveal the circumstances under which the system converges on rank-size as a steady state. Gan et al. (2006) show via Monte Carlo simulation that the law is a statistical phenomenon that does not require an economic theory. Batty (2006) describes the rank-size distribution as emerging as the self-organized steady-state of a complex adaptive system (Batty, 2006), and Corominas-Murtra and Solé (2010) describe the law as a common statistical distribution displaying scaling behavior, an inevitable outcome of a general class of stochastic systems that evolve to a stable state somewhere between order and disorder.

2.5. Contemporary contentions

The steady state argument has not been heard by the ''New Economic Geographers'' however. Their attention has been focused on the validity of Gibrat's Law and whether city sizes are better described by the lognormal or Pareto distribution. Michaels et al. (2008), using sub-country data for the US from 1880 to 2000, reject both Gibrat's Law and the idea of a stable population distribution. Garmestan and Allen (2008) argue that distinct regional city size distributions result from variable growth dynamics. González-Val (2010) and González-Val and Lanaspa (2011) conclude that Gibrat's Law holds only as a long-run average over time in the upper tail of the distribution, with size affecting the variance of the growth process, and Benguigui and Blumenfeld-Leiberthal (2011) assert that Zipf's Law describes only one of three distinct types of city size distributions that are currently observable.

These criticisms focus on urban growth paths over long spans of time, however, not on whether Gibrat's Law applies to the distribution of growth rates of sets of cities observed at a point in time, as is typical in much rank-size research, or whether the lognormal or Pareto distributions are better fits to the data.[11] Eeckhout (2004) kicked off the latter debate with two assertions: that Zipf's Law holds (i.e. the upper tail is Pareto) and that city growth is proportionate

[8] Nicholas M. Gotts states (personal correspondence 23 April 2004) that the missing assumption is that $ln(N) \gg C$, where N is the number of cities, and C is the ratio between mean and minimum city sizes, and because the mean size grows without limit, $C \to 0$ as $t \to \infty$. However, the Gabaix (1999, pp. 749–750) proposition 1 argument is couched in terms of normalized city sizes; i.e. his term *S-min* is fixed in relation to the expected sum of all city sizes (note 14 on p. 743), so the absolute minimum size increases over time and *S-min*/*S-bar* (where *S-bar* is mean normalized expected city size) remains fixed. Thus, Blank and Solomon's criticism is at least partially mistaken although Gabaix's argument for a slope of -1 does not hold in general, since it depends on the number of cities (N) being very large in relation to the size ratio between the mean and smallest cities (C).

[9] Other economists have focused upon other assumptions that are required to produce Zipf's Law, for example De Wit (2005).

[10] Axtell (2000) had argued earlier that there are three circumstances in which agent-based modeling may be deployed. The first is when equations can be formulated that completely describe a social process, and these equations are explicitly soluble, either analytically or numerically. A second is when mathematical models can be written down but not completely solved, in which case the agent-based model can shed significant light on the solution structure. Thirdly, when writing down equations is not a useful activity, agent-based computational models may be the only way available to explore process systematically. Axtell and Florida (2001) is an example of this third usage.

[11] Another criticism is that in OLS may be an inappropriate estimator. Gabaix and Ioannides (2003), following Embreachts et al. (1997), draw varying sized samples of city sizes from a population defined by an exact power with $\alpha = 1$ and demonstrate that the smaller the sample, the greater the underestimation both of the size of α (i.e. overestimation of q) and of the true standard error of the estimated coefficient. Nishiyama and Osada (2004) provide exact estimates of the bias and variance of q for varying n. Gabaix and Ioannides propose that Hill estimators be used as an alternative (Hill, 1970, 1974, 1975; Hill and Woodroofe, 1975), but they too are biased if the random growth model fails (Gabaix, 1999; Gabaix and Ioannides, 2003). Gabaix and Ibragimov (2007) offer a simple solution to correcting for the small sample bias; also see Terra (2009).

(i.e. Gibrat's Law operates), leading to a puzzle: Gibrat's Law gives rise to a lognormal rather than a Pareto distribution. Eeckhout argues that this dilemma results because so much research has focused on truncated distributions, such as census-defined metropolitan areas. Both distributions, he says track the data closely, but the Pareto coefficient increases as the truncation point increases on the horizontal axis and the Pareto fits the data less well, while that for the lognormal remains unchanged. In other words changes in the estimated Pareto coefficient are theoretically consistent with a changing truncation point of the lognormal distribution (Eeckhout, 2004, p. 1433).

To provide empirical confirmation he analyzed the distribution of the "complete" set of places in the US; i.e. all 25,359 legally-bounded places defined by the US Bureau of the Census in 2000, ranging in size from New York's 8 million to the smallest place with only 1 resident. The size distribution was shown to be lognormal and growth to be independent of size. The Zipf coefficient is nearly 1.0 if computed for metropolitan areas, but varies systematically with differing cut points on the complete distribution, consistent with the lognormal.

Levy (2009) challenged Eeckhout's conclusions, arguing that tests reject the lognormal for the larger cities in the upper tail, cities whose log size exceeds the average by three standard deviations. In reply Eeckhout (2009) argued that since both the lognormal and the Pareto distributions have tails with similar properties it is natural that the upper tail can be fitted to a Pareto distribution, even if the underlying distribution is lognormal. The difference between the two authors, Malevergne et al. (2009, 2011) argue, is that both Eeckhout and Levy use inadequate tests. After explaining how Gibrat's Law underpins both the lognormal and Pareto distributions (see footnote 5), they perform the UMPU (uniformly most powerful unbiased) test for the null hypothesis of the Pareto distribution against the lognormal and conclude that for the largest 1000 places in Eeckhout's data set the distribution is Pareto, confirming Levy's (2009) argument that the lower ranges of the data on places are lognormal but the top conforms to a power distribution.[12] Building on this, Giesen et al. (2010) propose that the double Pareto lognormal distribution (DPLN) is an even better fit. This distribution has a lognormal body but also features a power law in the upper and lower tails and arises from a stochastic urban growth process with random city formation.

3. Three questions

Stepping back from these debates, there appear to be three simple questions that need to be addressed empirically: What are the appropriate urban-regional units to which the size distribution models should be fitted? Do the growth rates of these units obey Gibrat's Law ? Does Zipf's Law apply strictly in the upper tail of the size distribution with $q = 1.0$? We provide answers for the US for the 1990–2010 time span.

3.1. The units of observation

Much of the debate about proper functional form stems from the New Economic Geographers' injudicious selection of units of observation. Legally-bounded pieces of real estate capture only pieces of regionally-integrated Economic Areas, with wide variations in size and rates of growth, and interdependence of growth rates within economic areas. The continued use of such units by, for example, Eeckhout (2004, 2009) and those building on his research adds confusion rather than clarity to the distributional question, because the fragmentation of legally defined entities adds significantly to the variance of growth rates and the length of the lower tail of the size distribution; his 25,359 places are parts of less than 200 spatially-integrated urban-regional labor markets.

The recognition that urban growth had to be understood within larger spatial units defined using labor market criteria led the US government to delineate Standard Metropolitan Areas in 1950. As useful as these areas were, their definition cut off many outlying areas tied by commuting to jobs located in urban cores and failed to recognize the significance of cross-commuting in the most densely settled parts of the nation, however. Therefore, after the 1960 census, a research team at the University of Chicago was commissioned to analyze the commuting data and to map the actual extent of the nation's commuting regions (Berry et al., 1968; Berry, 1973; Berry and Gillard, 1977). The Office of Business Economics (now the Bureau of Economic Analysis) of the US. Department of Commerce used these maps as the first step in the creation of a set of economic regions as they tried to develop regional accounts that summed to national accounts. In sparsely populated parts of the country the commuting data were supplemented by information on newspaper and wholesale markets. The resulting Economic Areas were seen by regional economists to posses distinct advantages: they completely disaggregated the United States into subregions, using county units as building blocks, and they had a high degree of "closure" with respect to their job and housing markets and the tertiary sector of their economies. This meant that forecasts based on assumptions about the economic base (job market) could be translated into population forecasts, income earned in each area could be equated with income spent plus saving, and a tertiary or "residentiary" economic sector could be identified whose growth, according to traditional economic theory, ought to be related to total sales made within the areas, in contrast to the primary and secondary sectors whose growth was related to sales made outside the Economic Area. The definitions were reassessed in 1977, 1983, 1994 and 2004, using the commuting information provided by the 1970, 1980, 1990 and 2000 censuses, resulting in some modifications but with the same goals and outcomes. Some areas were grouped because of increased cross commuting, and new regions were defined that centered on small economic centers in the less densely settled parts of the country. We believe that these spatial units capture the essential requirement of any analysis of city sizes and growth: the independence of their labor markets and therefore of the units of growth, satisfying Rosen and Resnick's (1980) call for use of integrated economic units. None of the spatial units proposed as alternatives (Holmes and Lee, 2009; Rozenfeld et al., 2010; Ye, 2006) satisfy this requirement. In the analysis that follows we use the EAs with populations exceeding 500,000 as the observations. At smaller sizes there is much greater variance in the growth rate depending upon features of limited numbers of export industries, whereas above 500,000 average growth rates converge on the national average.

3.2. Gibrat's Law for the Economic Areas

A simple cross-sectional test is sufficient to determine whether Gibrat's Law describes the growth in population of these Economic Areas. If the regression $\log p_{it} = \alpha + \beta \log p_{it-1} + \varepsilon_i$ is estimated and $\beta = 1.0$ Gibrat's Law holds. If the ε_i are *iit* normal the size distribution is Pareto (Malevergne et al., 2009, 2011).

For the period 1990–2000 $\beta = 1.004$ and for 2000–2010 the value is 1.015. In neither case can the hypothesis that $\beta = 1.0$ be rejected. The ε_i in both cases are *iit* normal, and the R^2 in both time periods exceeds 0.99. The conclusion is that Gibrat's Law holds, that the distribution is Pareto, and that Zipf's Law should apply strictly with $q = 1.0$

[12] Bee et al. (2011) argue that a maximum entropy test may be superior to UMPU estimates. Goldstein et al. (2004) propose use of maximum likelihood methods, but combine them with Kolmogorov–Smirnov tests.

3.3. Zipf's Law for the Economic Areas

We therefore next compute $log\, p_{ir} = log\, p_i - q\ log\, r_i$, where the intercept $\log p_i$ is the size of the largest Economic Area predicted by the equation. For 2010 the result is $log\, p_{it} = 7.82 - 1.009 log\, r$ with an $adj.R^2$ of 0.962. For 2000 and 1990, respectively, the similarly powerful equations are $log\, p_{it} = 7.760 - 0.994 log\, r$ and $log\, p_{it} = 7.693 - 0.986 log\, r$. In none of these years can the hypothesis that $q = 1.0$ be rejected. Zipf's Law applies strictly.

4. A problem in the uppermost tail

Despite the satisfaction of both Gibrat's and Zipf's Laws a problem remains in the uppermost tail of the distribution, however. If the rank size equation for 2010 is used to predict EA populations there is underprediction of the sizes of the largest 7–8 regions as shown in Table 1, with the differences increasing with higher rank. Others have noticed this upper tail problem. Rosen and Resnick (1980) suggested that the rank-size rule may be only a first approximation to the distribution of city sizes because they detected a statistically significant presence of the nonlinearity. Similar issues were raised by Vining (1976), Black and Henderson (1999), Dobkins and Ioannides (2000), Ioannides and Overman (2003), and Favaro and Pumain (2011), with the discussion focused on lack of independence of the units of observation either via intercity migration or due to some other type of spatial autocorrelation.

We suggest an alternative that we term the Megalopolis Hypothesis. Recall Gottman's argument (Gottmann, 1961, p. 5) when he wrote.

> "We must abandon the idea of the city as a tightly settled and organized unit in which people, activities, and riches are crowded into a very small area clearly separated from its nonurban surroundings. Every city in this region spreads out far and wide around its original nucleus; it grows amidst an irregularly colloidal mixture of rural and suburban landscapes; it melts on broad fronts with other mixtures, of somewhat similar though different texture, belonging to the suburban neighborhoods of other cities."

He postulated that there were clustered networks of urban regions that he called "Megalopolitan Areas," giving them such fanciful names as Boswash, ChiPitts and SanSan.

Following in Gottman's footsteps, we hypothesize that the linear fit in the uppermost part of the rank-size distribution requires recognition of several megalopolitan-scale clusters ("Boswash," "Sansan," "Chicago-Milwaukee," "Detroit-Cleveland," and "Dallas-Austin") formed by clustering 25 closely interdependent EAs.[13]

Recomputation of the Gibrat equation with the revised data set for the time period 2000–2010 produces $log\, p_{it} = -.48 + 1.013 log\, p_{it-1}$ with an $adj.R^2$ of.99 and a root MSE of.003. The 95% confidence limits for β are 0.997–1.03. Gibrat's Law holds; see Fig. 1. The Zipf equation for 2010 is $log\, p_i = 7.729 - .986 log\, r_i$ with an $adj.R^2$ of .99 and a root MSE of 0.03. The 95% confidence limits for q are −1.01 to −.97. Zipf's Law holds in the strict sense. See Fig. 2. The actual and predicted uppermost area populations are set down in Table 2.

5. Conclusions

Theory and empirical evidence converge. Gibrat's Law holds for the size distribution of U.S. urban regions, as does Zipf's Law in the strict sense. The fitted rank-size relationship confirms that the largest urban-regional units are megalopolitan in scale and when megalopolitan regions are included in the model the rank-size distribution maintains linearity throughout, without the uppermost tail problem of rank-size fits using the EA alone.

[13] The constituent EAs are: Boswash: EAs 22,49, 70, 72, 118, 127, 137, 174; Sansan: EAs 61, 97, 140, 145, 146; Chimil: EAs 9, 32, 43, 101, 108, 156; Cledet: EAs 13, 42, 87; Dalaust: EAs 35, 47, 166.

Table 1
Actual and fitted EA populations in 2010.

Economic area	Actual, '000	Fitted, '000
New York–Newark–Bridgeport, NY–NJ–CT–PA	23,290	66,437
Los Angeles–Long Beach–Riverside, CA	19,716	33,014
Chicago–Naperville–Michigan City, IL–IN–WI	10,531	21,930
San Jose–San Francisco–Oakland, CA	9715	16,405
Washington–Baltimore–Northern Virginia, DC–MD–VA–WV	9279	13,098
Boston–Worcester–Manchester, MA–NH	8341	10,897
Dallas–Fort Worth, TX	8073	9328
Atlanta–Sandy Springs–Gainesville, GA–AL	7630	8152
Philadelphia–Camden–Vineland, PA–NJ–DE–MD	7047	7239
Detroit–Warren–Flint, MI	6915	6509

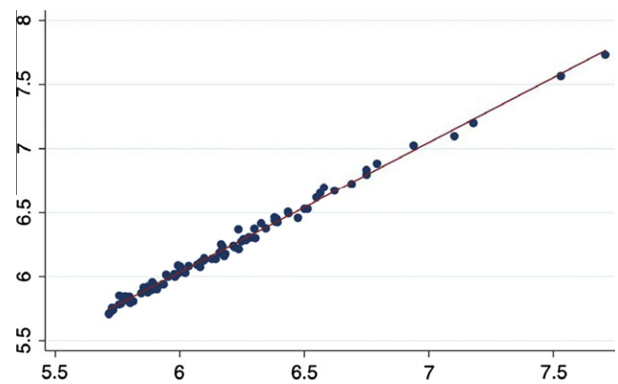

Fig. 1. Log of EA and megalopolitan population in 2010 (vertical axis) plotted against log of population in 2000 (horizontal axis), with regression line shown.

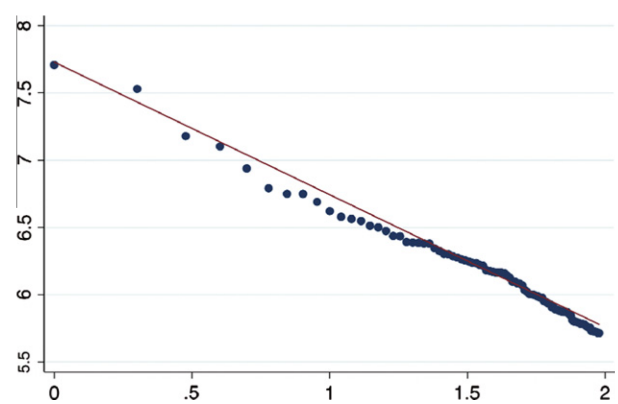

Fig. 2. Log of EA and megalopolitan population in 2010 plotted against log of rank.

Table 2
Actual and fitted megalopolitan area populations in 2010.

Megalopolitan area	Actual, '000	Fitted, '000
Boswash	54,587	53,617
Sansan	36,892	26,808
Chicago–Milwaukee	16,329	17,872
Detroit–Cleveland	12,518	13,404
Dallas–Austin	10,567	8936

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