11th November 2011

The Alonso Memorial Prize Lecture

Complexity in Regional Science

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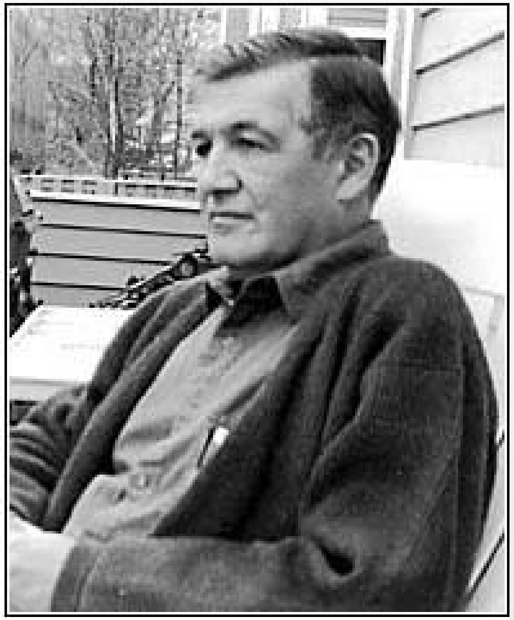
Bill Alonso was a revered name when I was a graduate student in Manchester in the late 1960s.

His book ***Location and Land Use*** (Harvard University Press, 1964) was the first formal statement of how urban structure resulted from the demand for and supply of land. His theory built on von Thunen's 1826 spatial market theory but he cast it in micro economic terms, building on the edifice of utility theory established in the previous 100 years.

I never met him personally but in 1970, at the European Regional Science Association meetings, held at the London School of Economics, I had the privilege of hearing him speak.

I am honoured to receive this prize for my book ***Cities and Complexity*** (MIT Press, 2005).



William Alonso, 1933-1999

68 PAPERS OF THE REGIONAL SCIENCE ASSOCIATION, VOL. XXVI, 1971

TENTH EUROPEAN CONGRESS OF THE REGIONAL SCIENCE ASSOCIATION

THE ECONOMICS OF URBAN SIZE'

Reprint No. 81

by William Alonso*

1. INTRODUCTION

At least since Aristotle, men have wondered about the best size for cities. In the last decades developed and developing nations, capitalist and socialist, have increasingly adopted more or less explicit policies on urbanization with special reference to city sizes. Most typically, these policies assume that the big cities of the nation are too big, and therefore try to disperse growth. Complementarily, in recent years such dispersal policies, and policies addressed to distressed or backward regions, have recognized that these alternative centers must be of a certain minimum size, however ill-defined, in order to be viable. In its simplest sense, the question of urban size consists of symmetric parts: how big is too big? and, how big is big enough?

Papers in **Regional Science**

A THEORY OF THE URBAN LAND MARKET

William Alonso

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Papers in Regional Science

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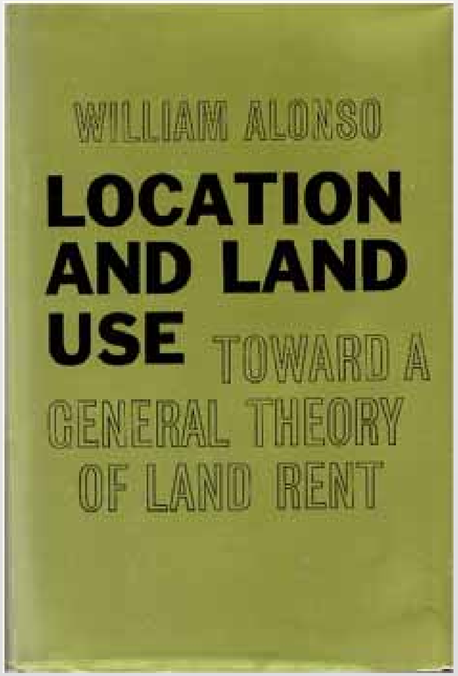
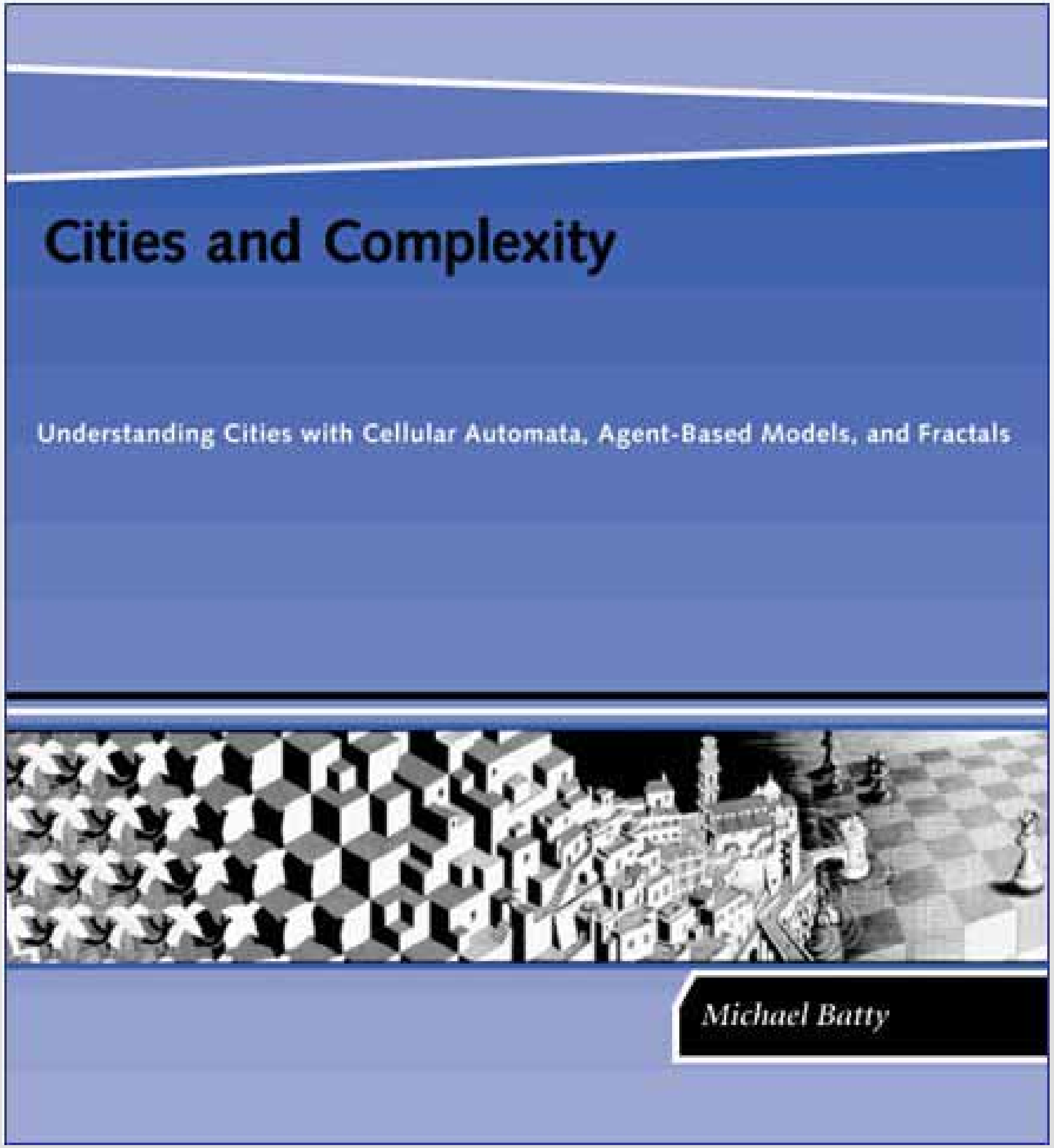


Harvard University Press, Cambridge, MA, 1964



MIT Press, Cambridge, MA, 2005

My Key Topics

- What Is Complexity? A Chronology and History
- Properties of Complex Systems: Decentralisation, Hierarchy, Emergence, Path Dependence
- Physical Demonstrations: Urban Form and Function
- Size and Scaling: Four (more or less) Laws of Scaling
- Measuring Complexity: Spatial Complexity
- Two More Things Very Briefly: First Network Science
- Second: Dynamics: New Styles of Model
- Conclusions

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What Is Complexity? A Chronology and History

Ideas about complex systems go back to prehistory, but it was not until the 1920s that these ideas began to develop coherently in the systems approach

The convergence of biological systems thinking with engineering and control in cybernetics and OR led to formal statements after WW2. Rick Church noted some of this in his opening talk.

The key notion was that systems were ordered and the idea was to express this order in generic terms that could be applied to any ordered collections of ideas, objects

The Systems Approach which developed in the 1950s and 1960s found its clearest expression in areas where theory and practice was inchoate, poorly developed, somewhat arcane

Systems were conceived as being centrally ordered, composed of subsystems organised hierarchically, dominated by negative feedback which suggested they were in equilibrium for the most part, and subject to explicit control.

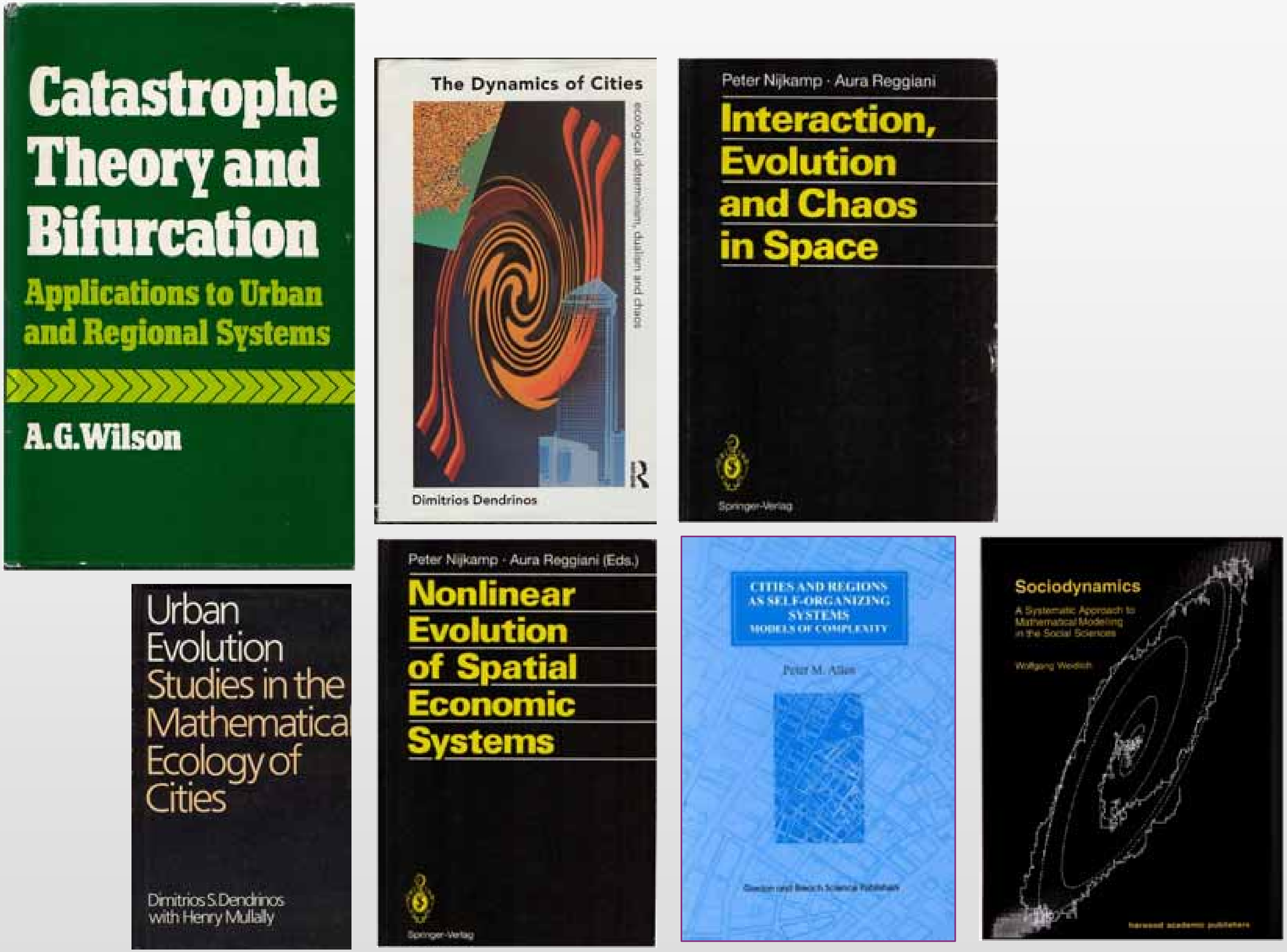
By and large, good candidates for such applications were things like cities and regions, that looked as though they might be in equilibrium, largely because their physical form was relatively inert and seemed unchanging for the most part.

But nothing could be further from the reality. We made the mistake of assuming that 'what we get is what we see'. Not so. Cities are never in equilibrium, they are constantly changing, they are dominated not by negative but by positive feedback. They are the crucibles of innovation. Their behaviour can be surprising, unpredictable.

Properties of Complex Systems: Decentralisation, Hierarchy, Emergence, Path Dependence

In essence, this movement gathered pace informally but then more formally in computer science where the idea of control from the bottom up came onto the agenda, in economics to an extent and of course in physics where notions of dynamics were being explored.

The notion of smooth change was quickly abandoned as ideas concerning catastrophe theory, birfurcation, chaos came onto the agenda. The notions of systems being far-from-equilibrium took hold. The notion of positive feedback is essential to these dynamics. Here are some contributions in regional science with people who are still very active


Catastrophe Theory and Bifurcation
Applications to Urban and Regional Systems
A.G.Wilson
The Dynamics of Cities
ecological determinism, dualism and chaos
Dimitrios Dendrinos
Peter Nijkamp · Aura Reggiani
Interaction, Evolution and Chaos in Space
Springer-Verlag
Urban Evolution Studies in the Mathematical Ecology of Cities
Dimitrios S.Dendrinos with Henry Mullally
Peter Nijkamp · Aura Reggiani (Eds.)
Nonlinear Evolution of Spatial Economic Systems
Springer-Verlag
CITIES AND REGIONS AS SELF-ORGANIZING SYSTEMS MODELS OF COMPLEXITY
Peter M. Allen
Sociodynamics
A Systematic Approach to Mathematical Modelling in the Social Sciences
Wolfgang Weidlich
harwood academic publishers

Much of the formal development of the idea that systems evolve from the bottom up has come from the Santa Fe Institute but the superstructure of ideas in this area is now much, much larger.

Essential to the notion of systems that are organised from the bottom up is that they evolve from their constituent parts, they grow and change, but most of all order and pattern emerge from the basic soup.

Emergence is key: it suggests no overall control, a limit on predictability, the notion that where we start matters – history matters – and that what ultimately comes about is dependent on the path we take – path dependence

Many of the old ideas in systems – hierarchy, interaction, subsystem structure – are still important to these concepts.

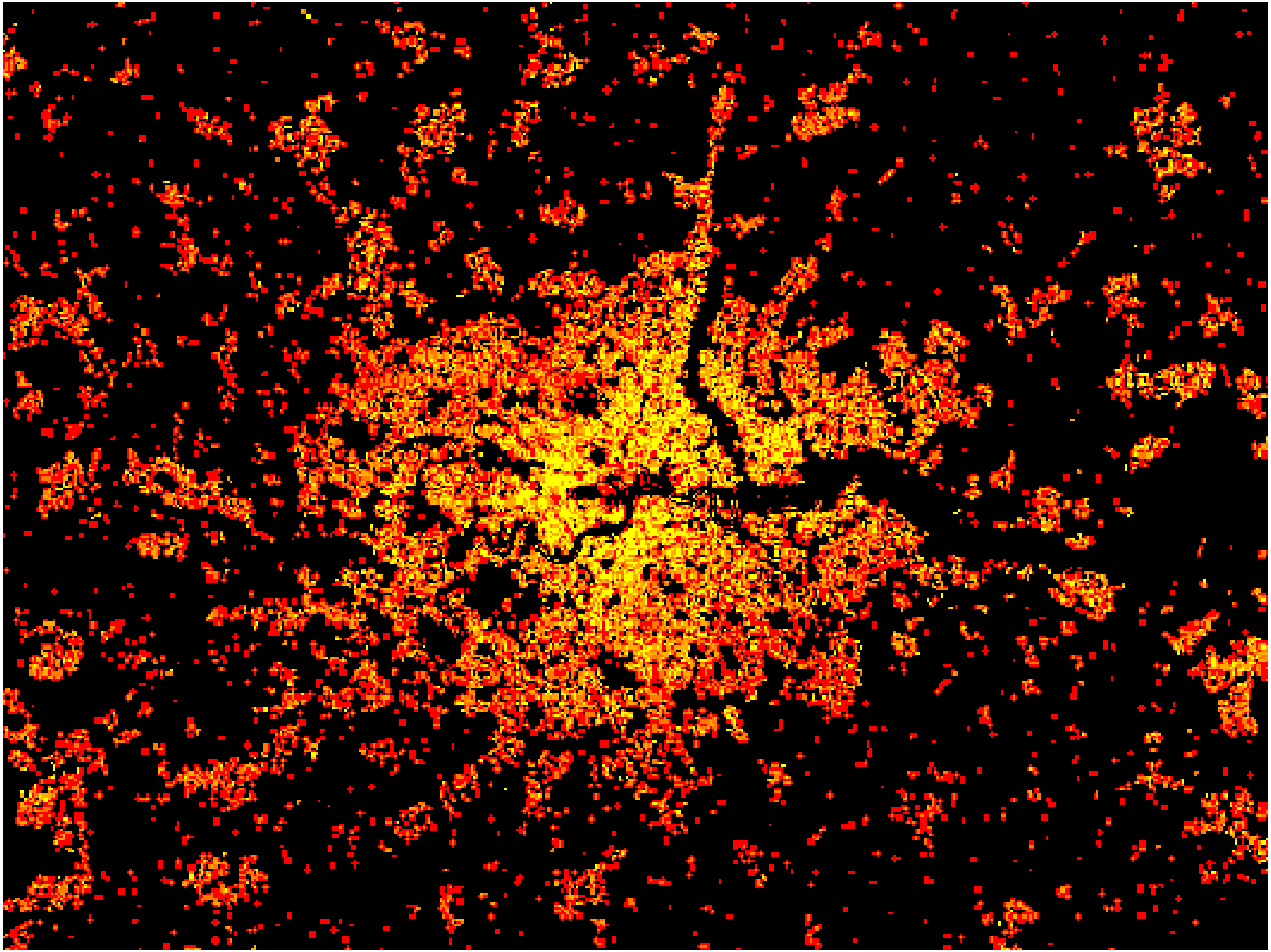
Physical Demonstrations: Urban Form and Function

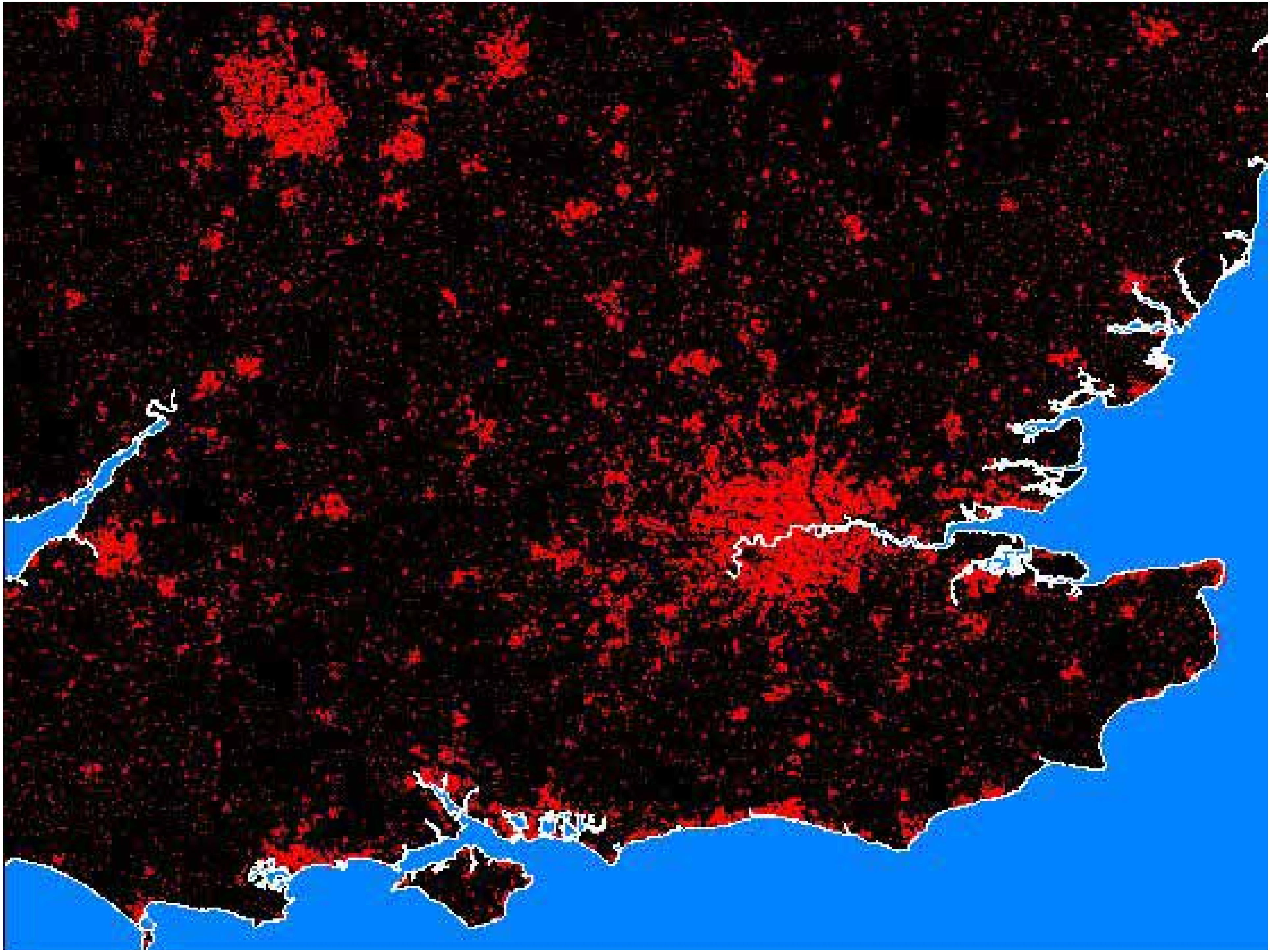
Ok. Let me try to demonstrate some of these ideas visually. One of the reasons why complexity theory emerged is because when we look at physical forms – patterns in cities – we see order and organisation and hierarchy that emerges from the bottom up.

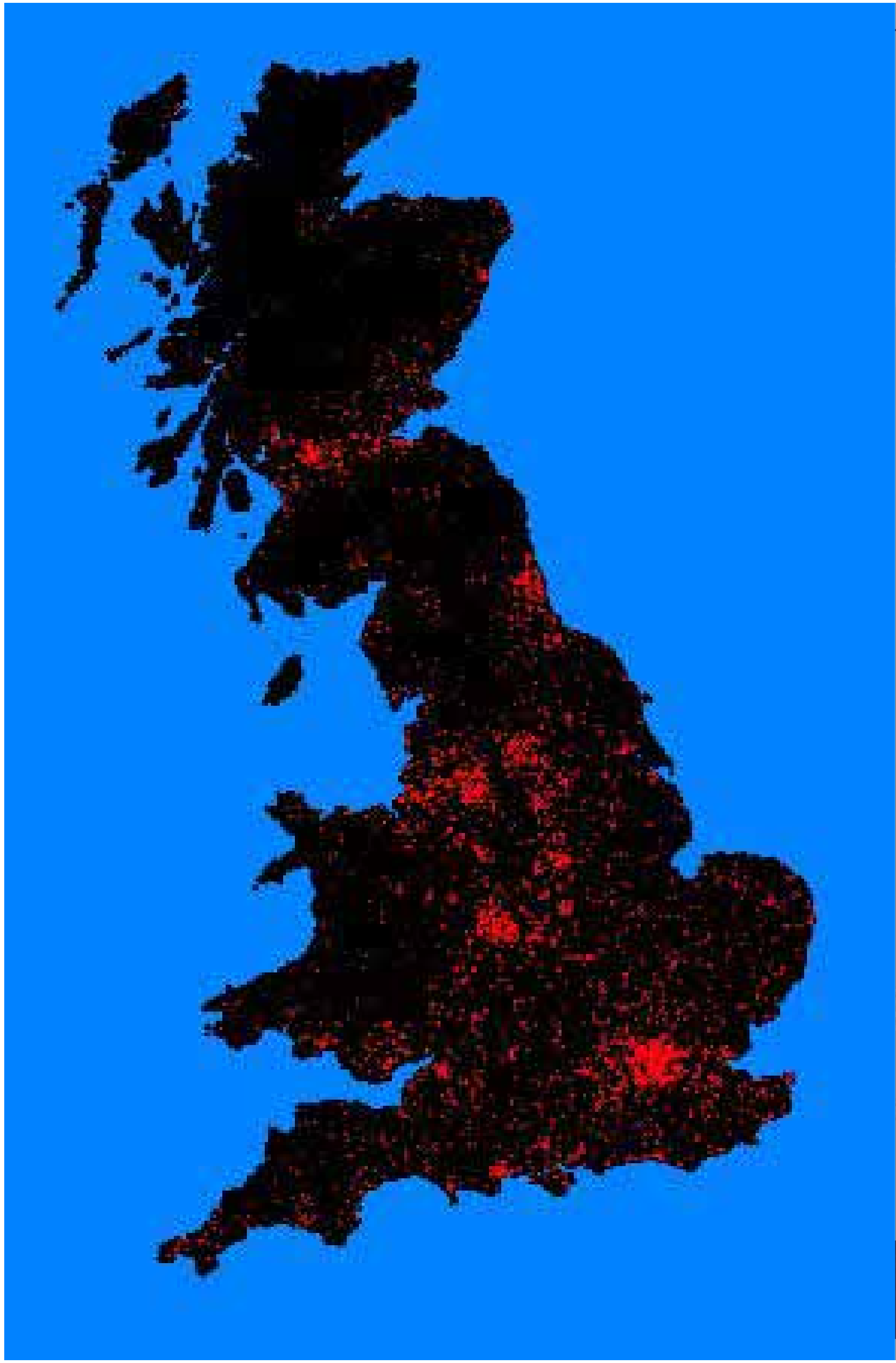
We see this best in nature really but we see it in cities rather clearly.

And yet the problem we have is that the dynamics of cities lies beneath these patterns and gives us a false sense of security that we can explain them

Here are some examples from nature and cities

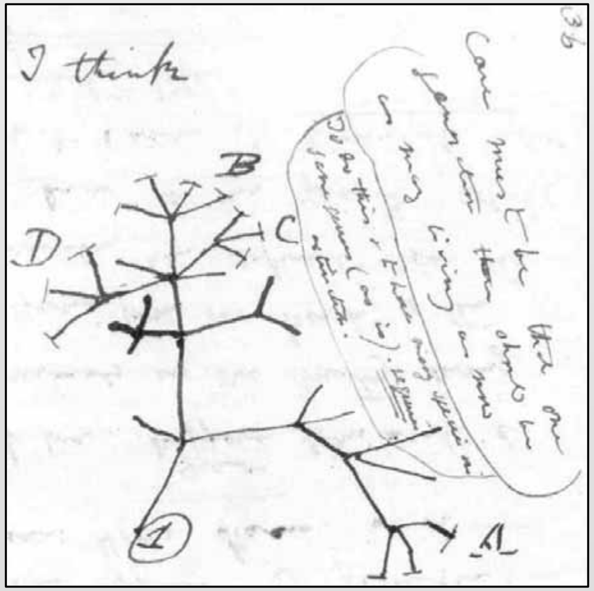


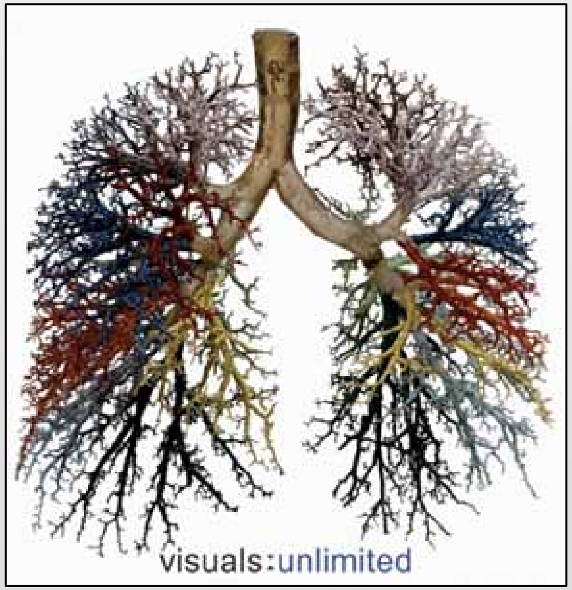


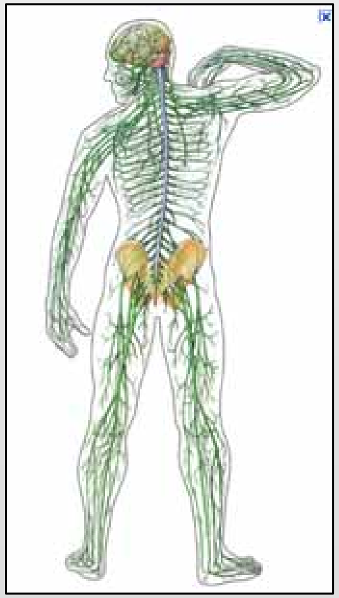


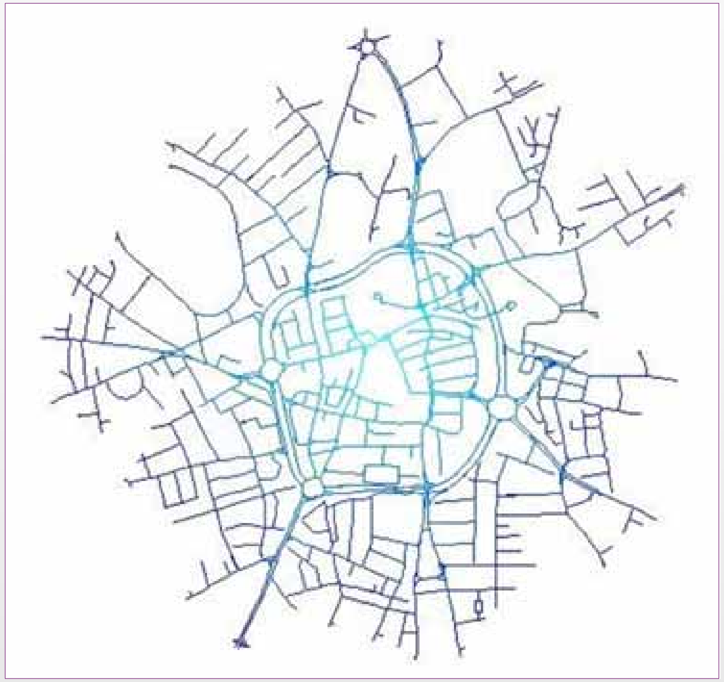

You can see size, and hierarchy, and scale in all of these – in fact these pictures are rather a good demonstration of scaling in city size – Zipf's Law but we are getting ahead of ourselves.

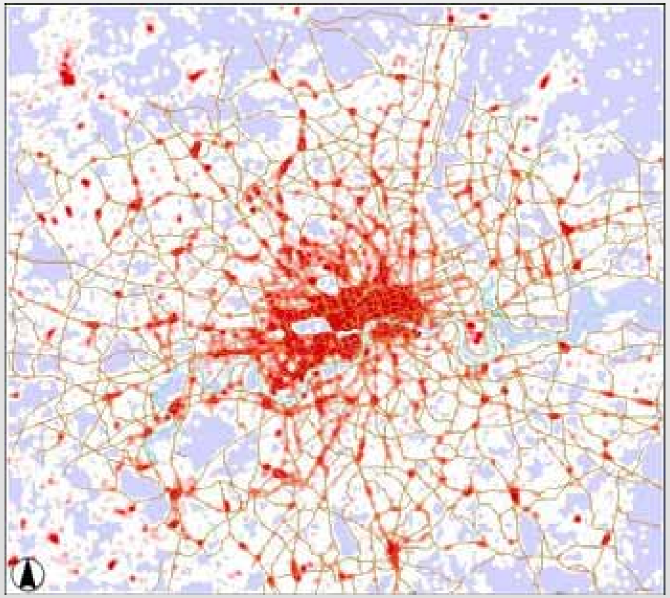
These patterns imply network structures that lie beneath and these are closely related to interactions. Here we can see the classic dendritic form that is key to the way nature delivers energy in an efficient manner to sustain its living systems

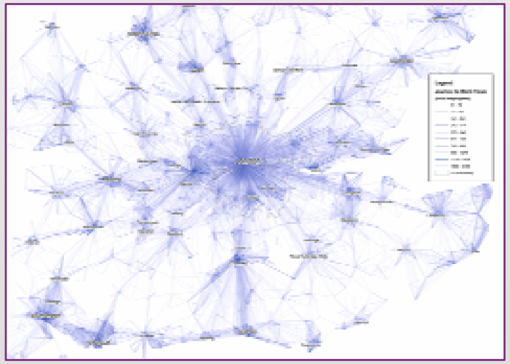












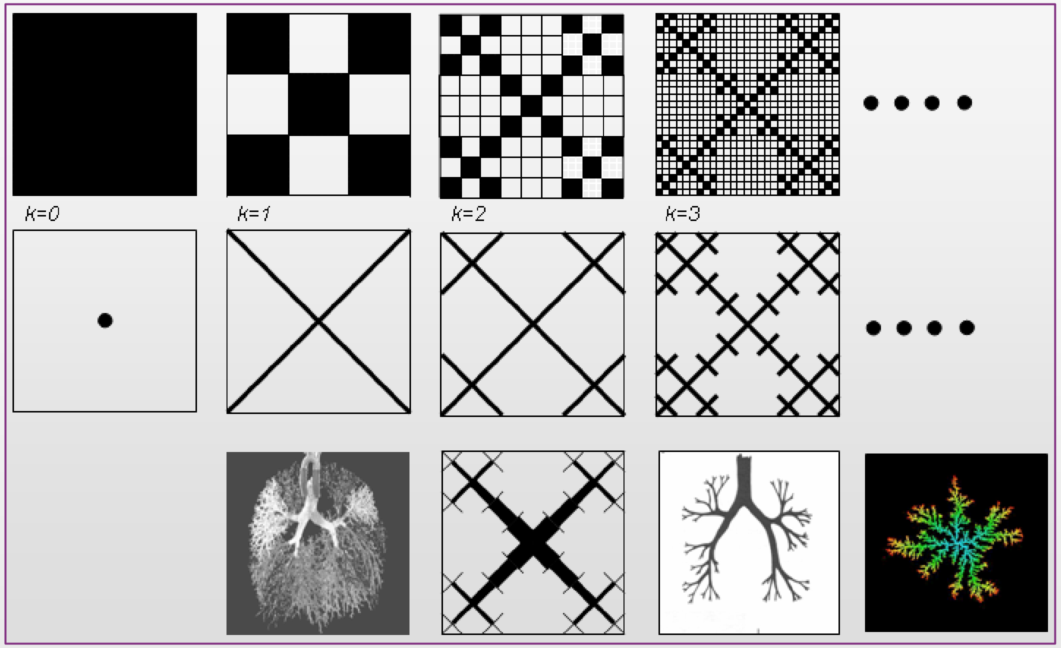
I could talk for hours about these sorts of patterns but let me say one more thing before I try and give you some sense of how these can all be tied together

Essentially the geometry that describes all these patterns is fractal geometry, first developed 40 years ago by Benoit Mandelbrot, and best seen in his wonderful book *The Fractal Geometry of Nature* (1977, 1982)

Fractal geometry throws up its own complexity – coastlines are infinite in length but the area they enclose is finite

The dimension of a fractal object is a real number not an integer and most of our world is in fact fractal – integer dimensions are the special case

But most important is that fractal objects scale – as you zoom in or out they look the same. They are self-similar. For example



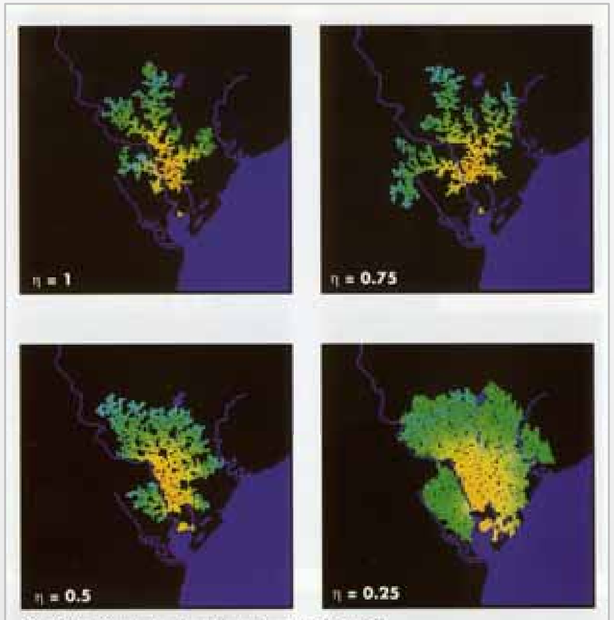
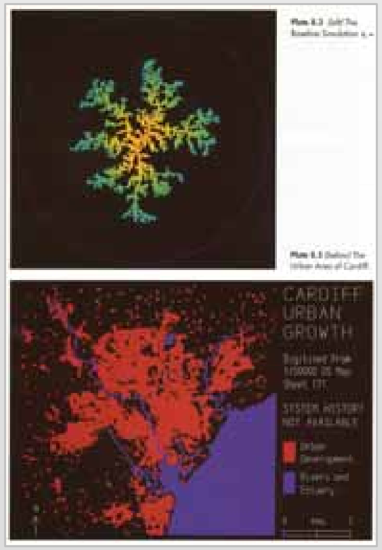
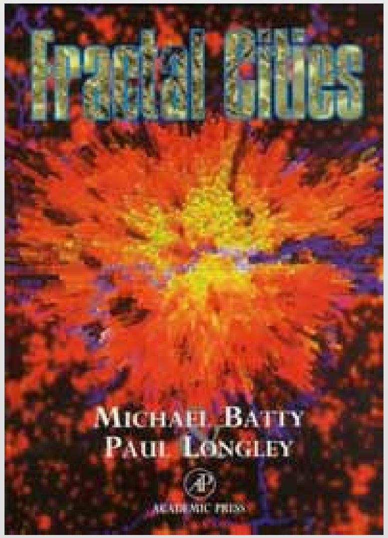


Plate 8.6 Simulating the Urban Growth of Cardiff





And this was the book that preceded my *Cities and Complexity* book that Paul Longley and myself wrote nearly 20 years ago

http://www.fractalcities.org/

Size and Scaling: Four (more or less) Laws of Scaling

Central to all this are ideas of scaling and these in the first instance are physically rooted.

The classic signature of scaling is a power law because it is the only algebraic function that has the same form when its scale is changed – change the scale from x to $2x$ and then the function changes from $f(x)$ to $2f(x) = f(2x)$

The only function with this property is a power law such as $f(x) = 1/x$

Now let me digress a little and say something about the growth of cities and what we know from complexity theory and scaling and fractal geometry so far. When cities grow we have observed several types of scaling – let me list these

As cities grow, they cluster more and more people together and the number of potential interactions grows more than proportionately

A city of P persons has *'potentially'* P^2 interactions, only a fraction of these can be realised of course but there is pressure to increase the number of interactions more than proportionately to population size

Cities change in scale and size as they grow – first there are less of them in terms of size.

Then their functions scale with size in terms of attributes of scale. This is what biologists call 'allometry' and economists call 'economies of scale' or 'agglomeration' economies

I won't make the links particularly here but a lot of what Gilles Duranton talked about yesterday in his address is related – I am not going to say consistent – with all of this

All others things being equal, ceteris paribus…….we can state the following about cities

- As they grow, the number of 'potential connections' increases as the square of the population (Metcalfe's Law, the network equivalent of Moore's Law)
- As they grow, the average time of travel inside them increases
- As they grow, the 'density' in their central cores tends to increase and in their peripheries to fall
- As they grow, more people travel by public transport
- As they get bigger, their average real income (and wealth) increases (West's Law) – *this is allometry*
- As they get bigger, they get 'greener' (Brand's Law)
- As they get bigger, there are less of them (Zipf's Law) – *this is city size – rank size*

In one sense, all of these are scaling laws – whether there are four or more is disputable but let me look briefly at the third of these observations: that is, as cities grow, the density in their central cores tends to increase and in their peripheries to fall

Yesterday Gilles stated that one of the outcomes of the von Thunen-Alonso-Beckman-Muth-Mills et al. monocentric city models was the notion that rents scaled inversely with distance (or travel cost) – as a power law – of course this is central to spatial interaction, so we really need to define a fifth law of scaling that says that *densities and rents decline as a power law with distance from their cores*

Provisionally I will call this Alonso's Law, especially as a little later in his academic career he wrote about movement in some rarely available but insightful papers such as this one ….

A THEORY OF MOVEMENTS: (I) INTRODUCTION

William Alonso

June 1976

Working Paper No. 266

This work is being carried out under grant #SOC74-24115 from the National Science Foundation, through the Institute of Urban and Regional Development, University of California, Berkeley. My special thanks go to the many people who have listened to me with patience over the past few years. Lyna Rossi deserves special mention for her help.

My point is of course that there are several scaling laws that pertain to cities and regions, and that these tend to be power laws – ranging from positive and negative allometry to inverse powers and so on

Whether they are negative exponential or inverse power can generate furious debate and there is substantial effort being put into ways in which power laws can be generated using simple models –

The heritage in this area is long and distinguished from Pareto, Yule, Lotka, Simon to Gabaix et al. so on fusing urban growth theory with random stochastic models in the Gibrat tradition, and these deal with city size (and firm size and so on)

And in terms of allometry from Huxley, Haldane and so on though to the Santa Fe group.

In spatial interaction from von Thunen to Alonso to Wilson and so on

And of course the economic tradition that Gilles focussed on.

Last but not least, much of this work in fractals and self-similarity came out of the quantitative revolution in geography from Garrison and Berry to Tobler, Getis, Nysteun, to Woldenberg and many others

Many of these people, if not most, have presented at these meetings over the last 48 years.

What I mean to show here is that despite the entangled nature of these scaling relationships, they do represent the signatures of complex systems, and there is much work of synthesis to do on a science of complexity in our field which makes it clear how we can reconcile them, one with another.

Measuring Complexity: Spatial Complexity

Any talk on complexity would not be complete without some sense of how we might measure it and in our field we have already flirted extensively with such measures in the early days when we used entropy and information to generate, yes – scaling models for spatial interaction.

Entropy is a good candidate for such a measure because it trades off size for distribution.

Shannon’s formula is the essence and can be stated as

$$p_i = \frac{P_i}{\sum_i P_i} \qquad \text{with entropy defined as } H = -\sum_i p_i \log p_i$$

As the number of objects n in the systems goes up, the entropy increases so this is a good property

But entropy also measures dis-order or dis-organisation with the assumption that a highly peaked probability distribution – ie where all the population live in the same place – shows that the system has little disorder – a lot of order – whereas a uniform distribution shows considerable disorder

Ok, I can't give you a primer on entropy but there is a lot going on in this area linking entropy to power laws – not simply in terms of deriving spatial distributions using entropy-maximising but actually measuring entropy per se

I had a go at this many years ago and more recently too and there are many extensions. Let me state my own version of spatial entropy where we add space explicitly

We now define the entropy of the probability density which is

$$\rho_i = \frac{p_i}{\Delta x_i}$$

We can simply take the expected value of the log of the inverse of this, that is the expected value of

$$\log \frac{1}{\rho_i} = -\log \rho_i$$

So the spatial entropy formula becomes

$$S = -\sum_i p_i \log \rho_i = -\sum_i p_i \log \frac{p_i}{\Delta x_i}$$

The spatial entropy formula has some very nice properties as it is really composed the Shannon measure and an expected size term as follows

$$S = -\sum_i p_i \log \rho_i = -\sum_i p_i \log \frac{p_i}{\Delta x_i}$$

$$= -\sum_i p_i \log p_i + \sum_i p_i \log \Delta x_i$$

This is the distribution and the number size effect in terms of n in entropy

This is the area size effect

Now I don’t have time to go expand all this but suffice it to say that I believe we need to develop real measures of system size and complexity and measure real places using these ideas. And show that as cities grow, they get more complex?

In this way we will get a better handle on the intrinsic structure of the systems that concern us.

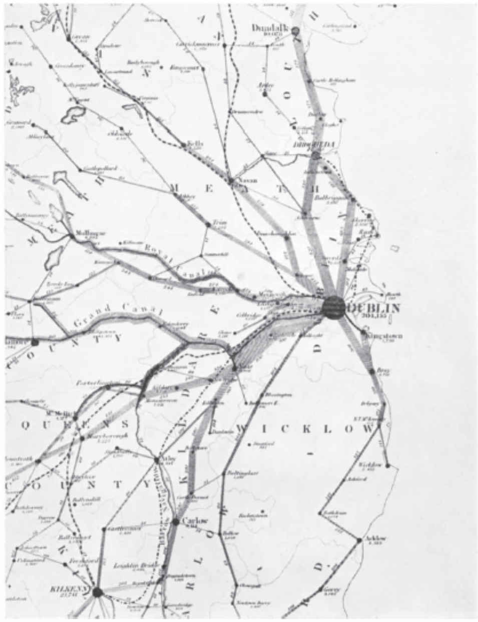
Two More Things Very Briefly: First Network Science

A talk on complexity would also not be complete without noting the remarkable development of network science and the kinds of scaling structures that dominate that field.

Barabasi is credited for noting that networks as graphs tend to be scaling in the size of their hubs or nodes and thus these are said to be 'scale-free'

There are other structures too such as small worlds and there is now a clear correspondence between how we might treat location in terms of the size of activities – summations of flows and summations of links. But there are many qualifications when it comes to spatial graphs – planar graphs

Work is beginning on the dynamics. Let me show some pictures.



Harness' Passenger Conveyance Map, 1837. Original, 1 inch to 10 miles: Brit. Mus.

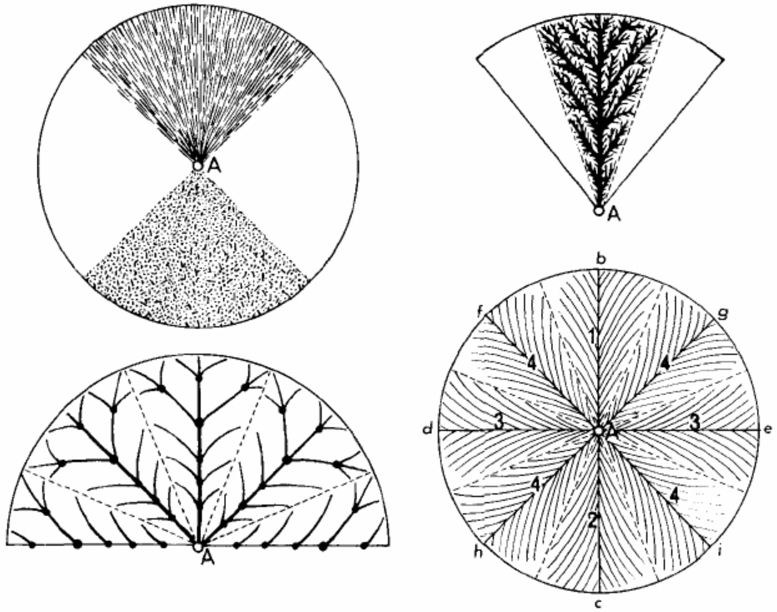
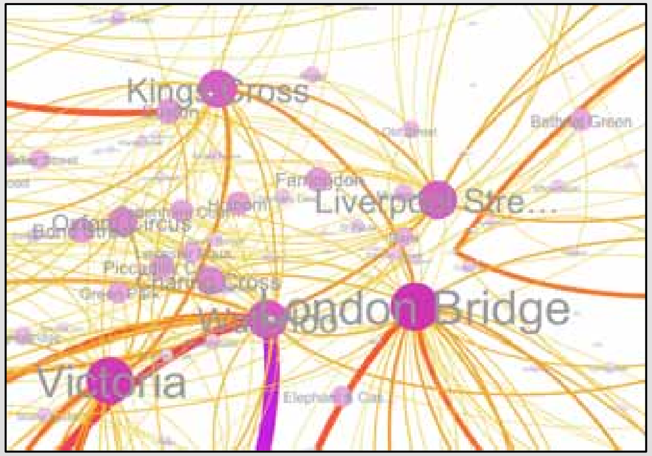
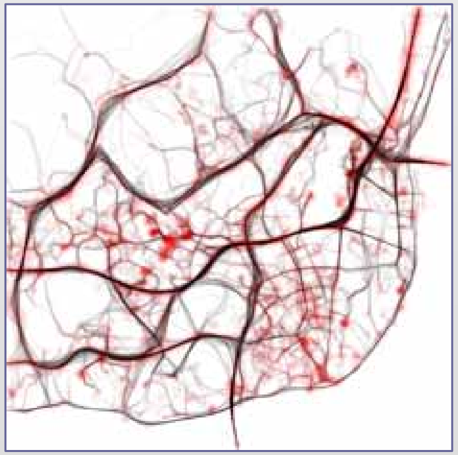


Figure 1.





Second: Dynamics: New Styles of Model

I mentioned dynamics before & a lot of new models of cities and regions have built on this – but some of the earlier ones have been rather aggregate. The quest to disggregate has been pursued ruthlessly and now there are new classes of model

ABM or agent-based models, microsimulation, cellular automata –focussing on very simple, often local dynamics generating global patterns emerging from micro decisions. Many models have tracked down scale where agents are individuals, traffic objects etc but some deal with groups and institutions.

Indeed the stream called geo-computation in this meeting is representative of these kinds of models. In a sense I showed some of these briefly before for our fractal models of cities.

Conclusions

Short and sharp.

- We need to disentangle the wood from the trees in terms of scaling
- Develop much stronger substantive theory of how form relates to function or how economy relates to physical space
- We need much better theory per se, that meets the strictures that Gilles Duranton made in his Presidential Address yesterday
- We need a wider synthesis based on notions about how networks deliver energy and information to cities and regions – we need to link flows to links – ***geography to geometry, and then geometry to economy, and back and forth.***

To conclude, I would like to refer you to my Web Sites

http://www.complexcity.info/

www.casa.ucl.ac.uk

and the book for which I received this award book, details of which are on my web sites