My Bit of the CASA Seminar, Wednesday, 5 October, 2011, on

Research Themes in CASA

Scaling, City Size, Allometry, Networks, Fractals & Simulation Modelling

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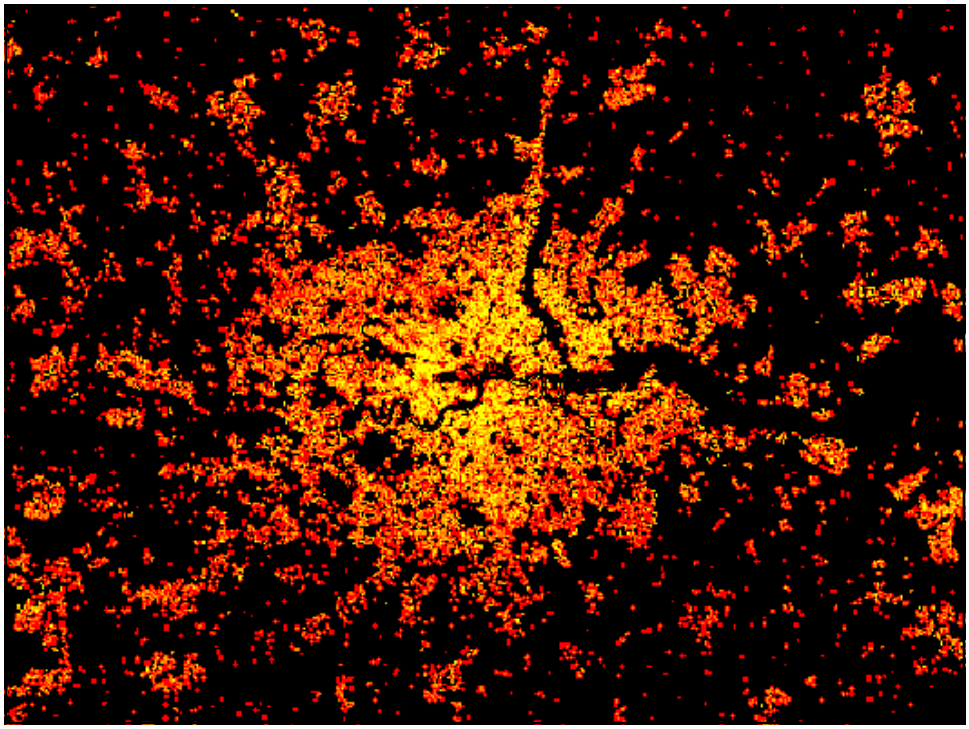
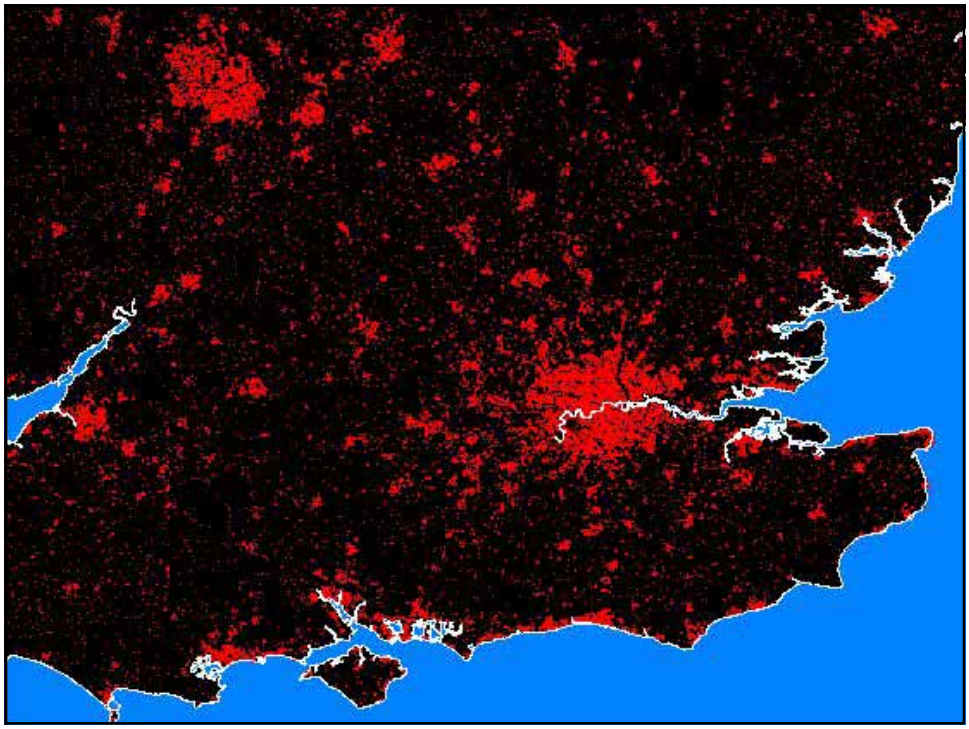
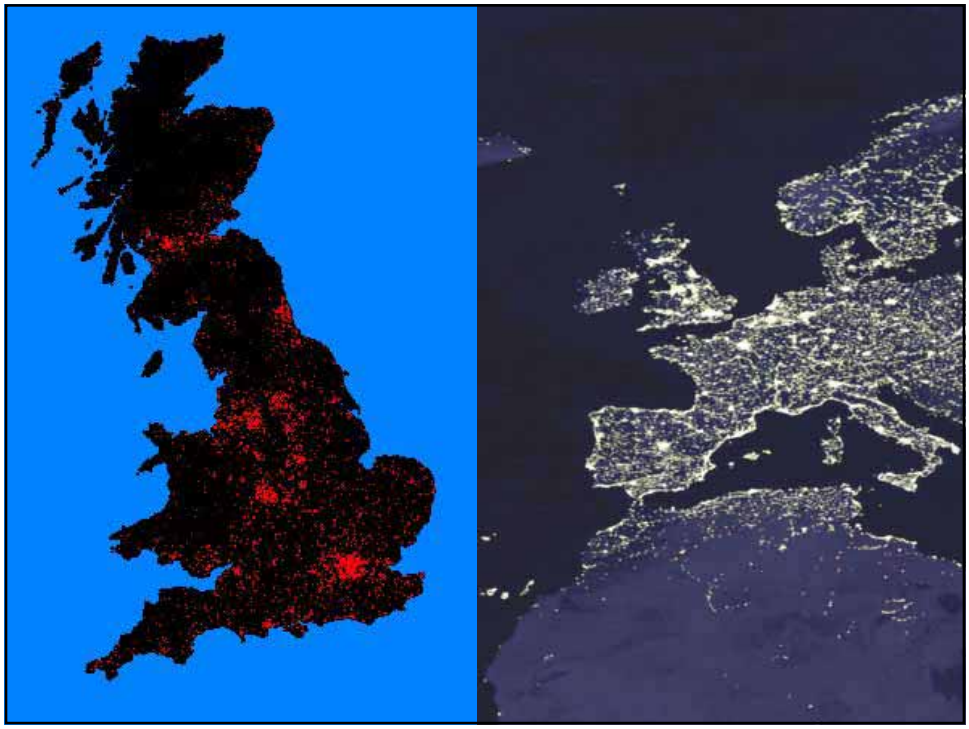
I am going to begin by telling you what **Scaling** is about because this underpins everything we do and certainly all the projects that I work with. This, as its name suggests, is about how things change in size – how they scale, quite literally

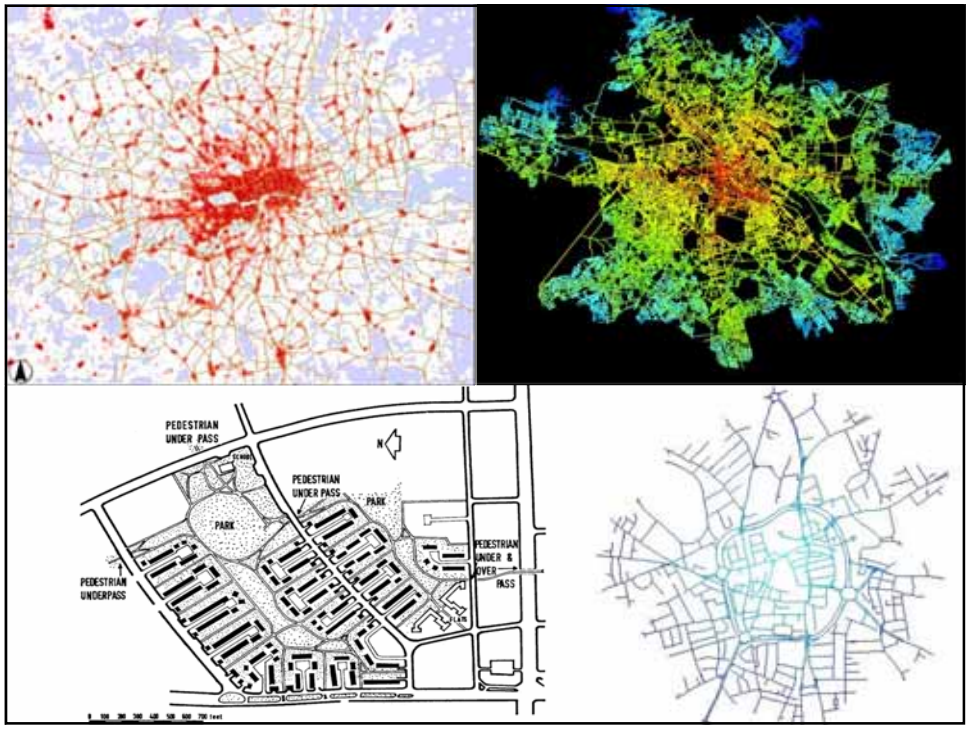
However in cities and many other social systems, there is ***incredible regularity*** in scaling.

Big cities look like little cities, big people look like little people, big firms look like little firms but with subtle differences. If you look at a city under a microscope, you see ever more similar (fractal) detail as you zoom in. Let me illustrate this for London





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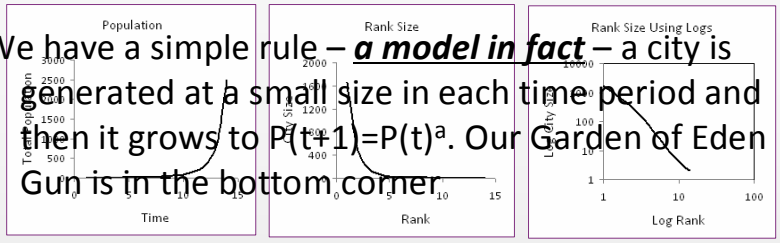
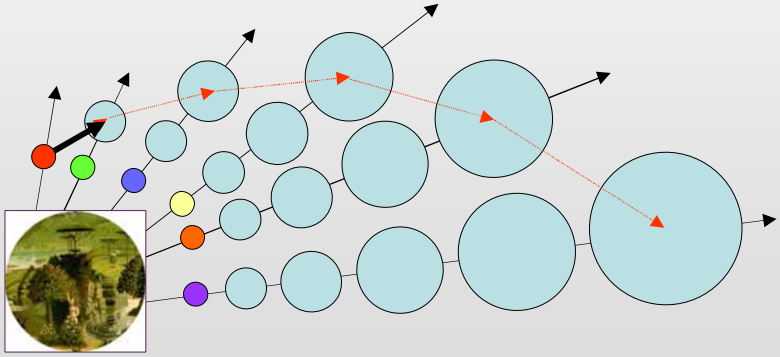
How does this all come about? And that relates to how we are researching it. Let me begin with a single small city.

Imagine a Garden of Eden which generates cities through time and as they are thrown out of the Garedn, they last and get bigger. This is a world where cities are continually borne, get bigger, never die. They get bigger exponentially as their growth rates are greater than 1.

OK, here is our Garden of Eden Gun that shoots out cities one after another

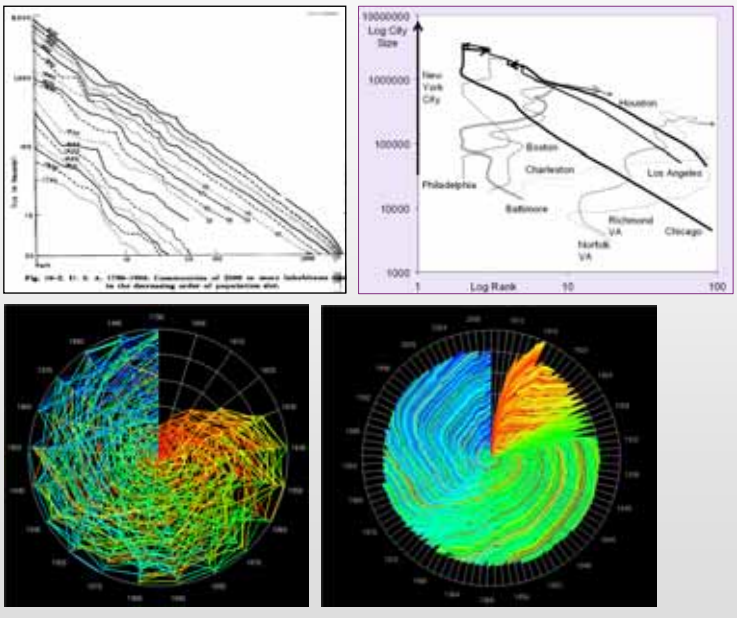


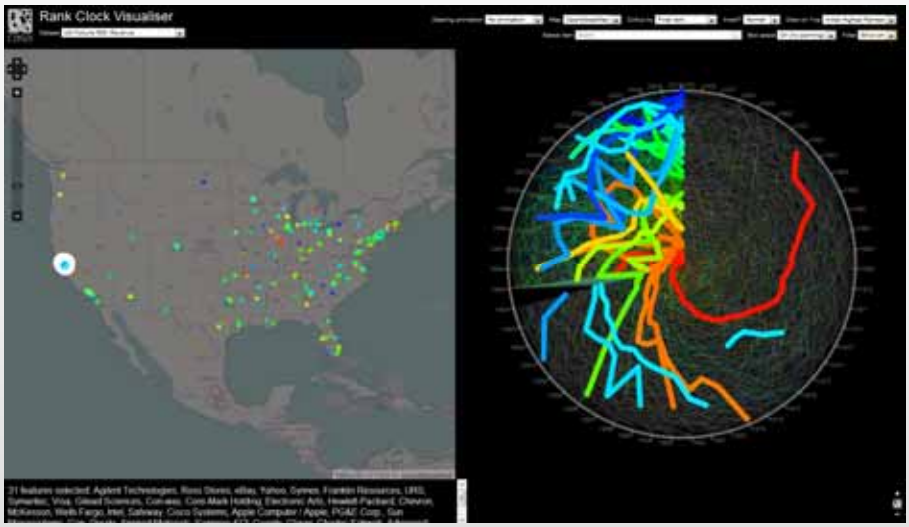
We have a simple rule – ***a model in fact*** – a city is generated at a small size in each time period and then it grows to $P(t+1)=P(t)^a$. Our Garden of Eden Gun is in the bottom corner



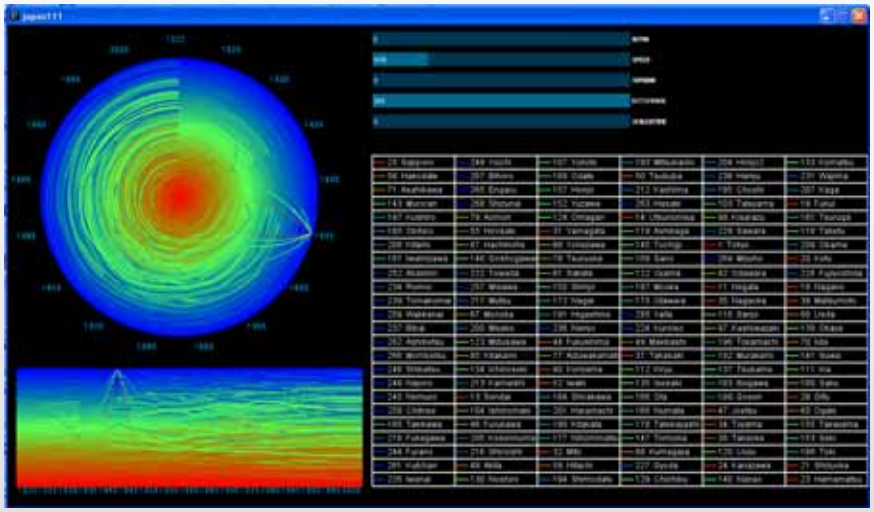
This is the **first sort of scaling** and we are looking at lots of systems using this kind of rank analysis. We have done it for many city systems and are doing it for firms, skyscrapers, network hubs in transport networks, global GDP by country and so on

Martin Austwick has some very nice visualisations of rank change that he has done using our rank clocks idea, Ollie O'Brien has implemented this in a web page context so it is interactive under our *Genesis* project, and I am doing some analysis of our Oyster Card data on London public transport with Jon Reades in our *COSMIC project* using scaling of the key hubs. Here are some images



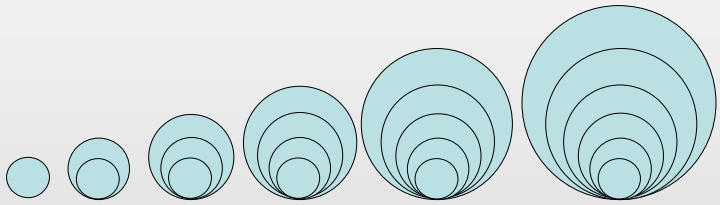


Ollie's Rank Clock Visualiser of Fortune 500 Firms



Martin's Rank Clock Visualisations – Japanese cities 1920-2010

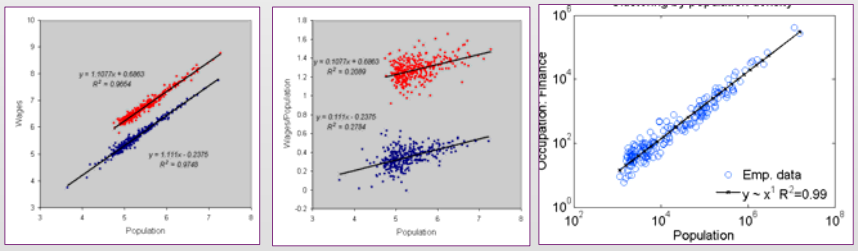
The second **sort of scaling** is when we look at the size of cities against other attributes – in other words the morphology of how they grow and this is called allometry. Back to our example.



P1< P2< P3< P4< P5< P6< ……..

As cities get bigger some attributes scale more than proportionately – positive allometry, some less – negative allometry. We express this as $P(i)=A(i)^b$

$P(i) \sim A(i)^b$ where if $b>1$ – positive allometry – we think of this as ***scale economies*** – another major concept in cities that comes from economics. OK, we are just starting some fascinating work on this in our *Mechanicity* Project – here incomes scale with pop in 350 US Metro Areas (SMSAs 1969 & 2008).



Log scales

There a third **sort of scaling** which is much more related to the sort of modelling that Alan Wilson has described and this relates to how the size of space which is measure by distance is explained.

We can illustrate this concept by noting that when people P get together in cities they generate contacts – and in principle if we can all reach each other, the number of contacts grows as P^2.

Now this is a bit like allometry –things that scale with city size are things that rely on contacts, but of course we are never likely to reach all of the people, so the power of P is likely to be a lot less than 2

So in a city we might reduce the scale of the interaction by finding an average distance d in a city that people are willing to travel to see each other

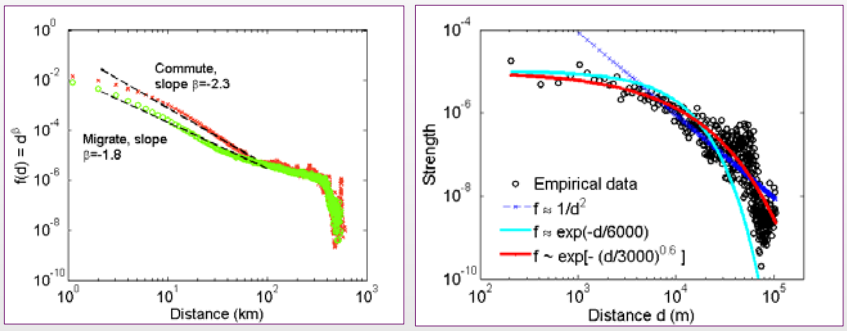
And then our number of contacts scales as (P x P) / d.

Now this formula works not only for the inside of cities but between cities. Lo and behold we get the gravity model if we work out the interactions T_{ij} between cities $T_{ij}=P_iP_j/(d_{ij})^c$ -c dimensions the model

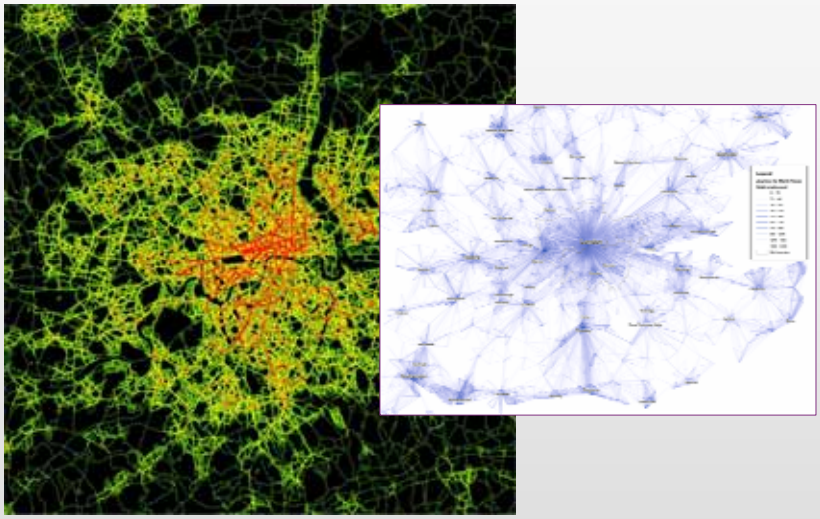
We are doing work on this as part of our project on a Unified Theory of Mobility in *Mechanicity* – which Anders, Elsa and Erez and Pete, but Joan and Jon from *Scale* and *Cosmic* are involved

There are three themes that spin off from this third kind of scaling – our unified theory that Anders is in charge of, our work on networks that Joan and Jon are in charge of (with a little from Anders), and our work on land use transport models that Camilo and Duncan are in charge of. This last project is ARCADIA that is still going on for another year and that is linked to SCALE. And Melanie will be involved in this.

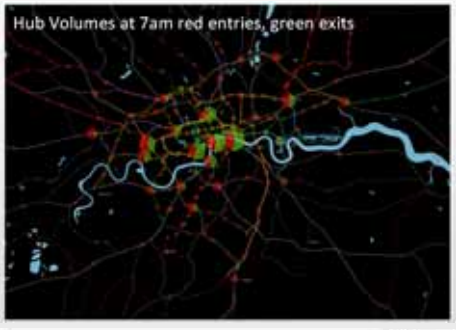
Let me give you some pictures of where we stand in these streams and first just show a couple of pictures that Anders and Duncan have produced

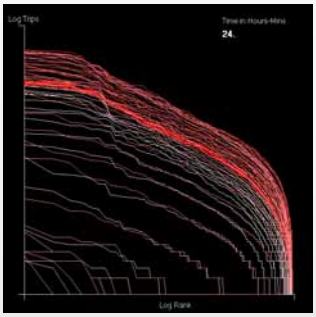


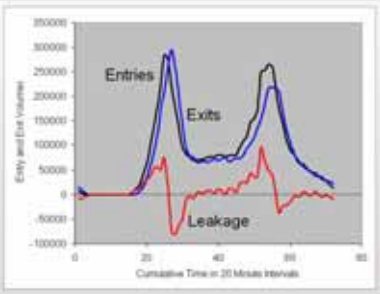
Once we are into interaction, then we very quickly move into networks and we are doing several things with networks that pick up on scaling, LUTI models and work with flows. Here is a typical network from Duncan of road accessibility.

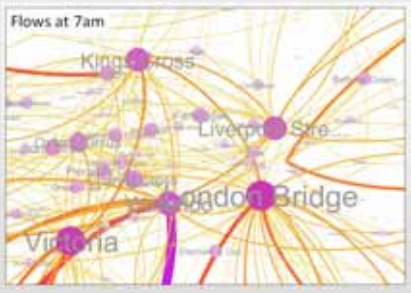


And from Jon's project with Oyster Card data

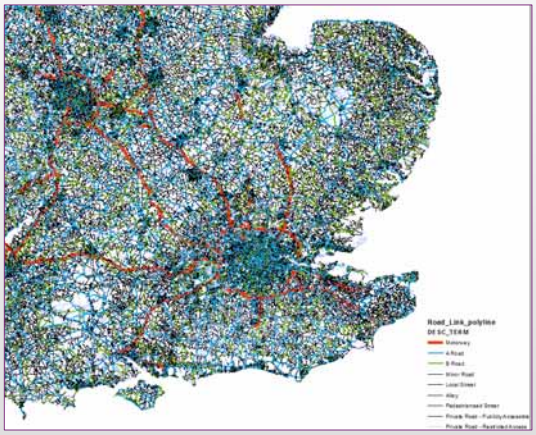








And from Joan's project building the networks from the SCALE and ARCADIA models



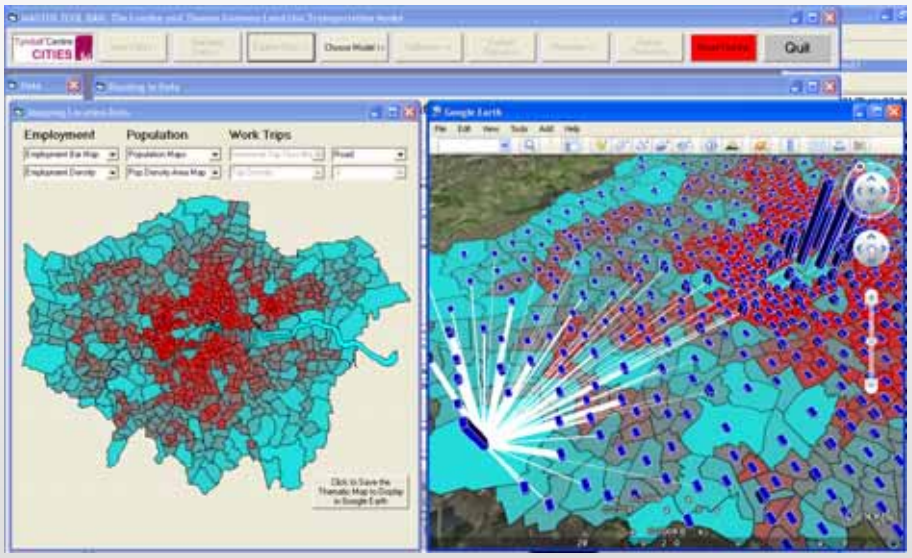
Let us now look at how we plug all this stuff into our models – how we fashion all this into building larger more integrated models.

I have built a model of Greater London for our Tyndall Project

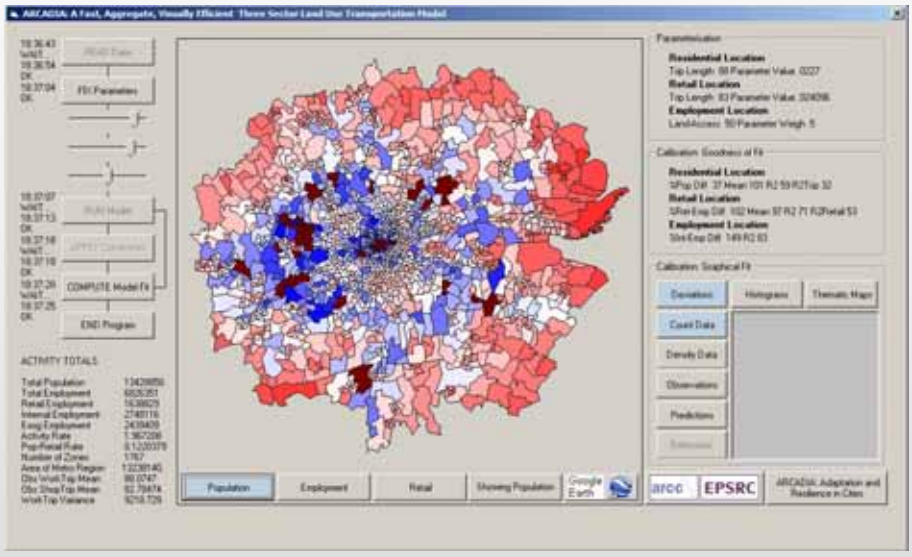
Camilo and myself and Joan are extending this with Duncan as part of SCALE and ARCADIA to the SE

Kiril is working on a Greater London cellular automata model as part of Genesis

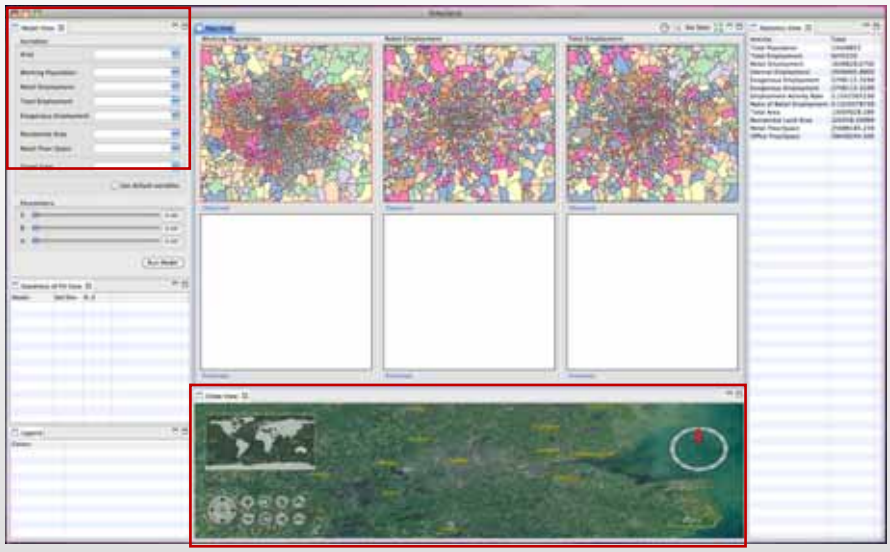
Erez is working up a model for housing in London as part of Mechanicity which is more agent based



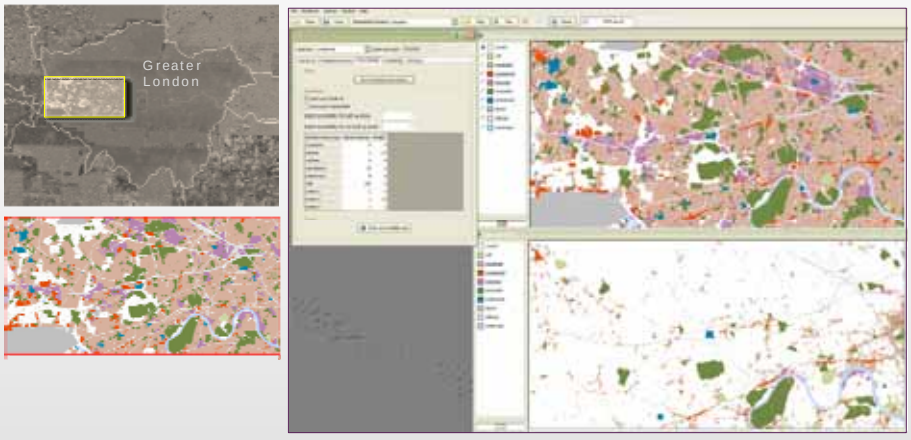
This is the Tyndall Cities Model for Climate Change



This is the Pilot Desktop Arcadia Model for London+SE

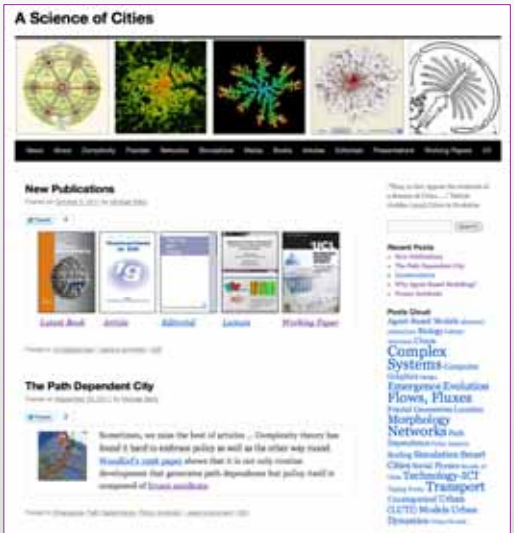


This is the Camilo's Web Based Arcadia Model



This is Kiril's CA Metronamica Model for West London which he is generalising to the whole metropolis under Genesis and we hope Talisman

Erez has only just come but he is working on scaling with the group in Mechanicity and here is his work on agent based models



I will post my bit under my recent Lecture post on www.complexcity.info

If there is time,

I will answer any

Questions

www.casa.ucl.ac.uk

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