

## The fractal simulation of urban structure

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Received 7 November 1985; in revised form 17 March 1986

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**Abstract.** This paper is addressed to the problem of developing realistic-looking patterns of land use and activity from predictions generated by conventional urban models. The method developed is based on a geometric model of irregularity involving hierarchical cascading and recursion, whose rationale lies in the emergent field of fractal geometry. First the idea of fractals—shapes with fractional dimension—is introduced and then it is shown how two-dimensional patterns whose dimensions are consistent with a large city such as London can be simulated at different levels of detail or recursion. It is then argued that the hierarchical structure of cities should be exploited as a basis for such simulation, and it is argued that discrete choice models of individual spatial behaviour have excellent properties which enable their embedding into such simulations.

The standard multinomial logit model is presented and then applied to house-type data in Greater London. A variety of models are estimated, house-type choice being related to age and distance from the centre of the city, and the spatial biases in these predictions are then mapped using prediction success statistics. These models are then used at the base level of a fractal simulation of house type and location in London. Random and deterministic model simulations are developed, and an unusual and possibly innovative feature of these simulations involves the way the inputs and outputs, data and predictions, are simultaneously displayed on the graphics device used. Conclusions for further research, particularly in spatial hierarchical modelling, are then sketched.

### Introduction

The authors of most urban models proposed so far have concentrated upon simulating economically orientated activities expressed in terms of employment, population, and transportation at the macrospatial level in large zones, or at the microlevel at the level of the individual or the firm. These traditions broadly follow research into models of the Lowry type in which spatial interaction is emphasised, or research into discrete choice theory where the emphasis has been on econometric estimation of individual choice behaviour. Although the two traditions are complementary and are increasingly being merged (Anas, 1982), urban models have rarely been proposed which take the level of simulation to the physical configuration of land use itself. One argument suggests that it is the activity level which is the appropriate level at which to simulate, for it is here that economic theory can be brought to bear on model design, and thus there is an implicit view that the translation of spatial activity into physical land use is a fairly trivial task or, at least, does not matter. It is more likely that the dearth of work in modelling land use per se is largely the result of unconscious neglect on the part of urban modellers who have found it easier to begin with activity simulation and whose disciplinary biases have constrained their interest in the physical form of cities.

However a major problem has begun to emerge in conventional modelling which relates to the spatial interpretation of model predictions. When model outputs in terms of activities are mapped spatially over aggregate zones or as individual point patterns, these outputs often look 'incorrect' in some indefinable physical sense.

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One can achieve exceptionally good model fits using conventional indicators, and such models can display robust and causally acceptable structure, but, when predictions are mapped, the whole does not seem to add up to the sum of the parts; systematic biases appear and the patterns often look physically unbalanced. In macromodelling, such biases can often be corrected or at least there are strategies which enable under- and overprediction to be handled consistently, but with discrete choice models the problem is that such outputs are rarely ever mapped spatially. Spatial bias is never assessed unless large-scale simulation is attempted, and thus in discrete choice modelling there are few checks on whether or not such models generate spatially acceptable predictions.

This is a general and difficult question which can be treated in a variety of ways, but a school of thought is fast emerging in the physical and biological sciences which suggests that the ultimate test of a model is that 'it must look right'. In one sense, this school represents a 'back-to-basics' movement which is based on dissatisfaction with conventional modelling strategies, but it also reflects the fact that computers are now so powerful that it is possible to present such general pictures of model performance quickly and efficiently. This approach to modelling is seen most clearly in the study of irregular spatial patterns whose irregularity has systematic properties which somehow must be captured. A new branch of physical form has emerged to deal with these questions which has deep and far-reaching implications for spatial modelling; in essence, shapes which are irregular but self-similar, such as coastlines, mountains, trees, crumpled newspaper, and a variety of physical phenomena, as well as artificial phenomena such as the rank-size distribution of cities, are treated. Such shapes are also considered to have fractional dimension (nonintegral dimension), this branch of geometry and the objects involved being called *fractals* by Mandelbrot (1982b).

It is in this area that a very strong case for judging the success of models by their appearance is being made. In our own area, it is easy to see that the physical properties of land use in terms of plot size, shape, and density seem to display the irregularity which is considered to be fractal. We know that cities are self-similar in a variety of ways, central place theory being the clearest demonstration of this principle (Arlinghaus, 1985). Thus the idea that city structures could be fractal is appealing, but of more import is the possibility that fractal geometry may well contain the basis for linking activity models to their physical context. This is the issue we will broach here. In essence, we will begin with traditional models of urban structure which reflect the spatial organisation of activities, and we will formulate these using discrete choice theory. We will then embed these into a fractal simulation of physical urban structure, thus enabling us to examine the spatial patterns and physical forms which emerge from model predictions of individual choice. At one level, this project is an exercise in generalising discrete choice models through large-scale simulation, but at another level, it is a project designed to increase the realism of spatial models.

There are many ideas introduced in this paper and it is important to be clear at the outset as to how these interact. Although the prime concern here is with the process of physical land-use simulation using fractal concepts, the issue of embedding traditional models in the framework of large-scale simulation is also important. In this we follow Anas (1982) in his concern for using discrete choice theory in a more general context. Several problems in discrete choice theory are also raised here, in particular spatial choice and aggregation, and these models are obvious candidates for further analysis in these terms. However we will not take these ideas forward here, but we will note their key implications.

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Second, we will introduce the idea that the model-building process is based on the loose cycle of inductive explanation, and deductive prediction. For example, discrete choice models are strongly inductive in their specification and estimation, whereas spatial interaction models of the entropy type are specified a priori, and are hence deductive. Proponents of either style of modelling rarely pursue the inductive-deductive cycle in any complete sense, but the argument here suggests that fractal simulation can provide a framework for such a process. Large-scale simulation itself establishes such a framework but there are few attempts which model the entire cycle. The work of Chapin and Weiss (1968) is an exception in that they attempted to explain urban growth using a linear statistical model and then reproduced that growth in a large-scale random simulation framework. This paper is very much in that spirit, but in our attempt to represent the entire model-building cycle, a number of corners will be cut and thus only picked up as items for further research.

A third issue involves the question raised originally of physical appearance. The patterns which we generate must 'look right' and this will be our guiding principle of fractal simulation. As we have a strong feel for what urban structure looks like from land-use maps, we will assess the appropriateness of the model estimation and simulation through appearance of the generated maps, as well as through more traditional statistics. Mandelbrot (1982a, page 581) says it all: "the basic proof of a stochastic model of nature is in the seeing; numerical comparisons must come second". This statement we will argue is as true of artificially generated phenomena as of nature, and in our quest to demonstrate this through computer modelling and simulation, computer graphics will be all-important.

A fourth issue relates to computer graphics in particular and computer modelling more generally. This work has only been made possible through advances in computer, systems and software. The project involved a remarkable mixture of computers, modelling systems, and styles. The discrete choice models, for example, are estimated using a standard logit package mounted on a mainframe computer, with intermediate processing on a minicomputer which acts as the front end to yet another mainframe on which the spatial mapping packages and data are mounted. The graphics simulation, however, is conducted using a graphics-based micro whose memory is mainly given over to the screen display. In fact, these styles are seen quite clearly in the figures reproduced in this paper in which the spatial predictions produced for the discrete choice models are presented using standard plotter outputs, in contrast to the simulations which are illustrated in photographs of the raster graphics screen.

We will now spell out the organisation of the paper. In the next section we will introduce key ideas concerning fractals—questions of geometry, self-similarity, and irregularity, as well as the development of computer graphics to display fractal shapes. It is important to present readers with a 'feel' for fractal simulation early on and thus, as soon as we get the idea of fractals across, we will demonstrate how computer graphics can be used to simulate fractal patterns. We will use a hypothetical city structure whose dimensions are those of Greater London (which will be our eventual application here too) and we will then show how we can simulate three spatial activities using ideas of recursion and hierarchy. The urban structure we capture is based on simple distance relations from the central business direct (CBD), and the simulation is structured randomly in the same manner as that used originally by Chapin and Weiss (1968).

At this point we will take one step back and briefly introduce the model-building process in terms of explanation and simulation, induction and deduction, emphasising the need to contain both within any complete process. We will then

explain how this process can be completed for the fractal simulation of urban structure in London. The inductive approach we will adopt is based on discrete choice theory, the estimation of a standard multinomial logit model (Hensher and Johnson, 1981) of housing choice, and measurement of its performance using McFadden's (1979) predicted success statistics. We will show how the model is fitted to data relating to choice of house type and location in London, as a function of key variables of urban structure relating to age and distance. Several models are fitted, some are reestimated, and computer maps are used to aid the interpretation process.

We are then in a position to begin fractal simulation of urban structure at whose base lie the fitted discrete choice models. The simulations are essentially visual, the data themselves being displayed on the screen and being replaced by predicted house types as the simulation proceeds. Two types of simulation are attempted—random and deterministic—and it is shown how it is necessary to develop deterministic procedures to enable the discrete choice models to generate realistic patterns. There are many conclusions to this paper and work both on fractal images (Pentland, 1984) and on spatial discrete choice models (Lerman, 1985) represent major directions for further research.

### **The modelling of spatial pattern**

#### *Fractal geometry: self-similarity and irregularity*

The classic way to introduce fractal geometry is to discuss the shape of a coastline. Viewed at a fixed scale, any coastline appears to have a degree of irregularity which can be measured using that scale. As one goes down-scale or nearer to the coastline if one is actually approaching it, the area in view gets smaller, but the scale also gets smaller and it seems the degree of irregularity is much the same as that viewed from the higher level. As one continues down-scale examining smaller and smaller nooks and crannies in the coastline, the level of irregularity always seems to be the same in terms of the scale chosen. Such shapes are said to have the property of self-similarity in that what appears at one level appears at the next level, and so on down the hierarchy.

There are several consequences of this fact. If the length of the coastline is measured between two fixed points at different scales, its length will increase as finer and finer scales are used. Unlike a straight line, a crinkly line seems to have infinite length because the crinkles exist at every level or scale of resolution. This conundrum has been noted for generations, but the first to give it formal expression was Mandelbrot (1967) who argued that such lines have essentially an undefinable length in that they are scale-bound or scale-dependent. But of more profound import was Mandelbrot's demonstration that such shapes have fractional, not integral dimension. A straight line has dimension 1, a plane 2, but coastlines have a fractional or fractal dimension between 1 and 2, mountains between 2 and 3, and so on.

Generalisation of the concepts of fractal dimension, and fractal shape is possible, although fractals do have some constant limit relating to their area (Mandelbrot, 1982b). Research into fractals has utilised computer graphics to display sections through fractals with greater than 3-dimensional form (Norton, 1982), and examples of fractals with dimensions between 0 and 1 exist based on phenomena which dissipate. In one sense, all phenomena are fractal because shapes with integer dimensions are special cases, and are abstractions in that perfectly integral shapes are impossible to reproduce in reality. Fractals thus appear everywhere and this is what makes the fractal idea so appealing, in that it is so obvious and real, yet so profound.

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Natural examples of fractals abound, but there are an increasing number of examples of man-made fractals, or rather man-made or artificial phenomena whose description is aided by the fractal concept. Cities are clearly self-similar in a variety of ways. Central place theory which is based on the regular subdivision of market-area size, leads to a distribution of market areas and central places which is consistent with the rank-size rule (Arlinghaus, 1985). This need not concern us further here, except it is worth noting that Mandelbrot (1960) has already shown that such size distributions are fractal. More recently, concern for processing images by computer derived from the development of remote-sensing has led to the analysis of spatial patterns which exist within such images. Fractal structure is clearly present in images of cities at a variety of levels, and Pentland (1984) suggests ways in which databases relevant to fractal simulation can be generated in this way. On a more prosaic level, simply examine a map of land use for a large city and compare this with other fractal scenes such as those based on terrain. A good map to look at which is consistent with the work reported here is Abercrombie's 1944 Greater London Plan Map which is reproduced on the front of *The Planner* (volume 70, number 11, 1984). Although land-use patterns have not been shown conclusively to be fractal as yet, enough evidence exists to encourage the use of fractal geometry in exploring their appearance. This, as we shall see, is essential to our purpose.

#### *Measurement and modelling*

Major efforts have already been mounted to classify fractals by their fractal dimension, and a variety of measurement techniques have been introduced. Several algorithms to determine dimension have been developed in computer cartography (for example, see Shelberg et al, 1982) and these have led to investigation of the stability of fractal dimension across different scales. This represents the first stage in the modelling cycle: determining whether or not the object and its shape are fractal, and then using this description as a basis for simplification through simulation. As part of this quest, fractals have been used extensively to enhance cartographic detail where a shape is too complex to describe in detail and where the shape in question can be approximated in its realism through fractal rendering (Dutton, 1981; Hill and Walker, 1982). The classic example is Australia whose basic shape can be encoded in eight pairs of  $x$ - $y$  coordinates and made realistic through fractal simulation along the lines linking the eight points (Fournier et al, 1982; Dell'Orco and Ghiron, 1983).

However, the momentum in fractal geometry is in modelling, rather than in description. Models of nonrandom fractals are based on strict rules of recursion which contain the principle of self-similarity, and the most widely used models of random fractals such as coastlines depend upon Brownian motion. Mandelbrot (1982b) shows how such stochastic models have the properties of self-similarity and fractional dimension, and these have been demonstrated as relevant models for terrain by Mandelbrot (1975) and Goodchild (1982). The key issue underlying a model of a fractal involves the degree to which form can be abstracted and simplified. In phenomena with a high degree of detail, where detail is self-similar with respect to different levels of hierarchy, and where the surface form is nondifferentiable, it is impossible to achieve anything approximating a *total* description, which in any case is a contradiction in terms. As Mandelbrot (1982b, page 201) says: "The goal of achieving a full description is hopeless, and should not be entertained". But realism of a kind is still required, and thus models become a necessity.

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Fractal-based description is thus centrally bound up with fractal simulation, once the principles of self-similarity and their application to generate irregularity have been identified. Models of fractal enhancement which introduce detail into straight-line segments are in this spirit, as are those introduced by Goodchild (1982) in the simulation of terrain. In cities, land-use patterns are formed from individual sites and parcels whose irregularity is conditioned from a myriad of historical, social, and physical instances. Such patterns are impossible to describe in detail and defy conventional modelling at a variety of scales, and would thus appear to be fractal. Consequently, fractal simulation is required in models which are attempts to generate such patterns. This is a critical point for, in an important sense, this paper is about using fractals to generate perceived realism in which traditional urban models can be embedded. This is a more modest goal than designing a fractal model of urban structure, for it implies that fractals are useful not only in generating the underlying processes of form but in rendering the forms produced from traditional models, thus making them visually acceptable.

#### *Simulation through computer graphics*

This last point brings us to the role of computer graphics. There are many instances in which a fractal kind of realism is required. Typically in movies a specific realism is not required and this is particularly true in science fiction films where the realism of the scene is tempered by one's imagination. There are some classic applications, for example, the pictures of mountainous terrain generated by Carpenter at the Boeing Aircraft Corporation (see Greenberg et al, 1982) and the pictures of the creation of a living planet from a dead world which forms the Genesis sequence in the movie *Star Trek II* (Smith, 1982).

A major boost to fractal research has come from computer graphics, for computers lend themselves to easy applications of fractal principles in areas such as cartography, biology, medicine, and geomorphology (Batty, 1985). Strictly, fractal landscapes and terrain are best generated using a model of Brownian motion, but this can be extremely time consuming. Shortcuts to such modelling exist based on exploiting the recursive structure of fractal surfaces in which a part 'looks like' the whole: this is self-similarity. A popular method involves continual subdivision of space in a constrained random fashion, and this is the method used by Smith (1982) and Fournier et al (1982) to construct their landscapes. The method has come in for considerable criticism from Mandelbrot (1982a), but it remains the quickest and most straightforward to implement, and it is used here. We will describe the method in the next section, but in essence it involves generating high levels of spatial resolution as viewed from a given scale, by successive subdivision of the whole space in triangular fashion. Other tessellations of the plane have been proposed and used (Herbert, 1984), but triangular subdivision remains the most elemental spatial generator.

We can now state the core problem to be addressed in this paper before we present a simple demonstration, prior to our fully fledged simulation. In essence, we are going to use a technique for generating realistic-looking images of the city by random subdivision of successive levels of the spatial hierarchy until a level of resolution is reached below which it is not possible to discriminate. At this lowest level, activity within this space is predicted using a traditional urban model. As such, what we are proposing is a new framework for conventional model simulation. In previous applications, such frameworks were either ignored or were unable to generate realistic-looking images with the obvious properties of self-similarity and irregularity.

## A demonstration of fractal simulation

### *Recursion and hierarchy*

The idea of modelling self-similarity at different scales involves finding a generating function which can be applied to each scale in a recursive manner. A simple example of recursion in a spatial system might involve a rule for generating central places of different orders: such a rule might involve market area, range of goods, population size, variety of consumer goods retailed, and so on, and would normally be applied first to the largest centre and then sequentially to lower order centres. The scheme in which such recursion takes place in spatial systems is usually hierarchical. We begin with the whole space or the terrain and subdivide it into a regular number of subspaces, quadrants say. Then each quadrant is further subdivided into quadrants and so on down the hierarchy until the level of resolution required is reached. Clearly the recursive rule involves locating lower order spaces through nonoverlapping subdivision. Such a scheme generates a hierarchy in which one space is subdivided into four, four into sixteen, sixteen into sixty four, and so on.

The hierarchy is an artifact of the method. It does not necessarily have any substantive meaning and this is clear from its use in the generation of computer-graphics landscapes where it is a simple device to control the level of fractal detail required. However, in the urban system, there is a possibility that the hierarchies used to generate land use could reflect notions of the city as an ordered system. Central place theory and neighbourhood hierarchies have already been mentioned, but hierarchies of traffic routes, public-sector organisations, and even firms exist if the original region is chosen to be sufficiently large. In one sense, treating cities as hierarchies is somewhat controversial, since in a number of studies, notably that by Alexander (1966), it is argued that hierarchy is too simplistic an ordering device and that activities and land uses in cities are composed into overlapping areas whose order is more lattice-like than hierarchical. However, this takes us to questions of the rationale for hierarchical organisation in cities which we must postpone until the next section. Here we will simply demonstrate the idea of a spatial hierarchy which is a consequence of the method of fractal simulation.

The method we have developed begins by dividing the original urban space, a circle centred on the CBD, into ten triangular sectors or segments. Each sector is then subjected in turn to hierarchical subdivision, and once the required level of fractal detail has been reached over any sector, the simulation moves on to an adjacent sector and subdivision begins again. The process begins with the eastern sector and rotates in a counterclockwise fashion until all sectors have been treated. We will not comment in detail on the appropriateness or otherwise of this prior geometrical constraint on the simulation. Suffice it to say that the radially concentric and sector structure of the contemporary city is reflected in this organisation, and that decomposition using a square grid would be less appropriate to highly polarised cities such as London. However, subdivision using quadrants or quadtrees may be preferable for more dispersed patterns. This is a matter for detailed research.

We will begin by first defining the spatial units or zones in question. The original circular space, referred to as  $Z$ , is subdivided into ten sectors, each sector referred to as  $Z_\theta$ , where  $\theta$  is an index reflecting the angular orientation of the sector. Within each sector, the zones are referred to by  $Z_k(r)$ , where  $k$  is the zone and  $r$  is the hierarchical or recursive level. From each branching of the hierarchy, there are  $s$  zones, thus  $k = 1, \dots, s$ . Over the levels of the hierarchy given by recursive levels  $r$ ,  $r = 0, \dots, h$ , particular zones are referred to using the sequence  $i, j, k, \dots$ , where  $i$  is a typical zone on level  $r - 2$ ,  $j$  is a zone on level  $r - 1$ ,  $k$  is a

zone on level  $r$ , and so on. The generating rule used to subdivide zones from one level of hierarchy to the next is given by

$$Z_k(r) = G_k[Z_j(r-1)], \quad j, k = 1, \dots, s, \quad r \geq 1, \tag{1}$$

where  $j$  is the zone being subdivided on level  $r-1$ , and  $G_k$  is the subdivision operator. A particular sequence of zones can now be generated in the following way. The process is begun by applying the rule in equation (1) to the original sector,  $Z_\theta$ ,

$$Z_i(0) = G_i(Z_\theta), \quad \theta = \frac{2\pi}{10}, \frac{4\pi}{10}, \dots, 2\pi, \quad i = 1, \dots, s. \tag{2}$$

Recursion in equation (1) using equation (2) leads to the sequence

$$Z_n(r) = G_n\langle G_m\{\dots G_j[G_i(Z_\theta)]\dots\} \rangle. \tag{3}$$

Because  $s$  zones are generated from each branch in the hierarchy, it is easy to show that at the  $r$ th level down the hierarchy there are a total of  $s^{r+1}$  zones. There is also a need for a stopping rule which ends the recursion.

In our case, we are subdividing to form a triangular mesh. The original segment  $Z_\theta$  is divided into four triangles in the manner shown in figure 1, and thus  $s = 4$ . From this diagram, it is clear that at recursive level  $r = 0$ , there are four sectors in the original segment; at level  $r = 1$ , sixteen; at  $r = 2$ , sixty four, and so on.

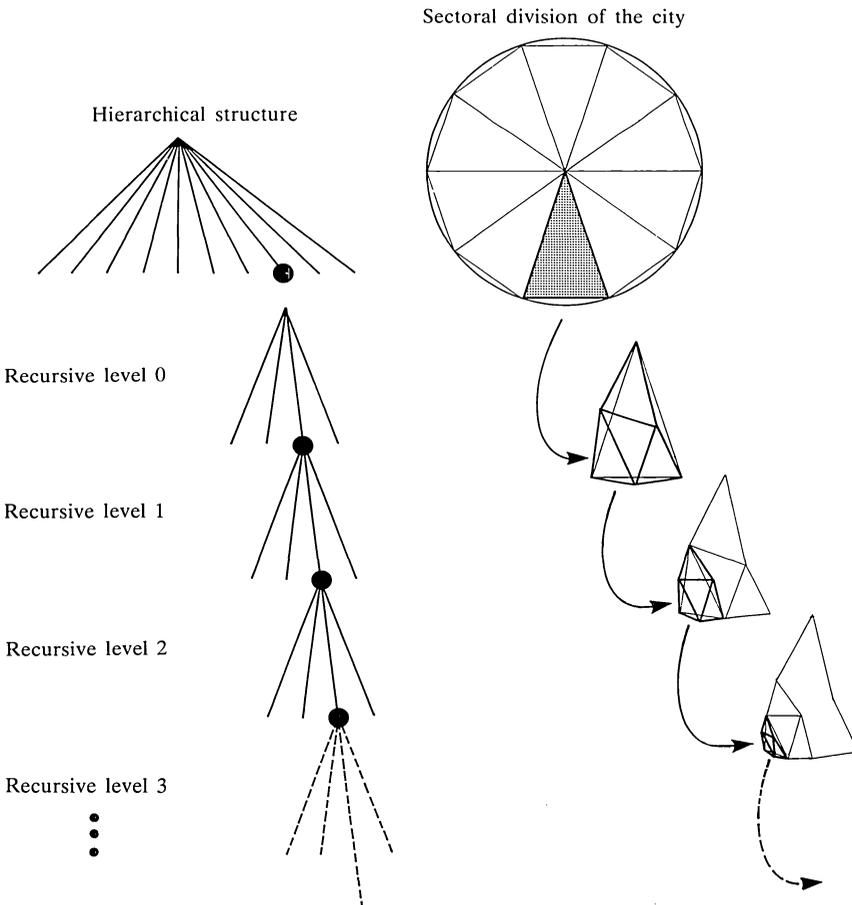


Figure 1. The recursive structure of fractal simulation.

The stopping rule is based on the level of resolution below which further spatial detail is not required. In this case, it is the level of pixel resolution of the display screen (which is  $650 \times 256$  pixels). A quick calculation shows that with ten sectors when  $r = 6$  we are below the level of resolution of the screen, and thus in the rest of the paper we will find that fractal detail can be most clearly articulated at levels  $r = 4$ , and  $r = 5$ , not greater.

We have chosen  $G_k$  to reflect the subdivision of a triangle space into four triangles in the manner shown in figure 1. This involves mid-point displacement of each side in a constrained random fashion, the degree of constraint reflecting the degree of irregularity, hence fractal dimension of the resulting surface. This is the technique used by Fournier et al (1982) which they argue is an appropriate approximation to Brownian motion in the plane. The algorithm used to effect the displacement uses simple trigonometric functions to compute the associated coordinate pairs which define the triangular mesh. The degree of randomness introduced is difficult to quantify in any simple way, but it is reflected in the displacements shown in figure 1.

One last point relates to the way the subdivision is effected. Strictly, as the mesh is generated and plotted, points adjacent to new subdivisions which are required in forming a contiguous web must be stored and recalled when required. However, this has not been possible with the graphics microcomputer used here and thus overlaps and crevices appear in the net where triangles are not 'glued' together correctly. In fact, a certain degree of control over this problem is possible and in the event it has not turned out to be too serious. Occasionally, fissure lines appear in the pictures generated which imply inaccurate 'glueing', but these are minimal. This is another problem which will be handled in future research when better graphics machines are available to the project.

#### *Simplified urban structure*

We have already introduced the idea that traditional models of urban activity are to be used to predict the activity type at the level of fractal detail realised. At present, these models are only used at the lowest level, not at intermediate levels which would imply that the hierarchy used in simulation has substantive meaning. Thus in the simulation, once a lowest branch in the hierarchy is reached, these models are invoked to enable activity types to be determined. Here we have assumed three key urban activities: commercial-industrial land use ( $l = 1$ ), residential-housing ( $l = 2$ ), and open space-recreational ( $l = 3$ ). The models for these activities are based on simple relations of distance from the CBD which establish profiles giving rise to concentric ring structures—the so-called Von Thünen rings—which characterise urban land use in strongly monocentric cities. In general, these profiles are structured so that commercial-industrial land use dominates the core of the city, residential-housing the periphery.

The general form of the model predicts a probability,  $P^l(d)$ , which is a function of distance,  $d$ , from the CBD, and specific to each activity,  $l$ . This is given as

$$P^l(d) = \alpha^l + \beta^l(d - \lambda^l), \quad l = 1, 2, 3, \quad (4)$$

where  $\alpha^l$ ,  $\beta^l$ , and  $\lambda^l$  are parameters whose magnitude and sign control the profile of the probability surface with respect to distance from the CBD. First for commercial-industrial activity,  $l = 1$ , equation (4) is written as

$$P^1(d \leq 400) = 1.38 - 0.0074d,$$

where the probability declines inversely with distance, touching 0 when  $d \approx 185$ .

When distance is greater than 400 (screen units), the probability is set at a minimum value given by

$$P^1(d > 400) = 0.002 ,$$

reflecting a minimum threshold on the existence of such activity. Comparing these two equations, we see there is a break in the profile from  $d = 185$  to  $d = 400$  where  $P^1(d) = 0$ . To control for this, an additional equation is also applied which is set up as the conditional

$$\text{if } P^1(d) < 0.04 \quad \text{then } P^1(d \leq 400) = 0.04 .$$

The combined effect of these equations generates the commercial-industrial profile shown in figure 2.

Residential land use ( $l = 2$ ) is controlled by a similar set of equations which reflect both positive and inverse distance relations. Then

$$P^2(d \leq 315) = 0.20 + 0.0024 (d - 30) ,$$

$$P^2(d > 315) = 0.88 - 0.0035 (d - 315) .$$

The effect of these equations is to produce a rising profile of probability from  $P^2(0) = 0.128$ , to a maximum of  $P^2(315) = 0.88$ , which then declines to  $P^2(561) = 0$ . To enable a minimum threshold for residential activity to be set, the conditional is introduced as

$$\text{if } P^2(d) < 0.05 \quad \text{then } P^2(d) = 0.05 .$$

Last, for open space ( $l = 3$ ), the relationship is one of inverse distance,

$$P^3(d) = 0.12 - 0.0002 d .$$

The probability declines from  $P^3(0) = 0.12$  to  $P^3(480) = 0$ . To ensure the function does not predict negative values, the conditional

$$\text{if } P^3(d) < 0 \quad \text{then } P^3(d) = 0$$

is invoked. These three profiles are shown in figure 2.

If we examine these probabilities, it is clear that these are nowhere normalised to sum to 1 exactly. We have done this so that  $\sum P^l(d) < 1$  ( $l = 1, 2, 3$ ), the residual probability is regarded as the probability of vacant land occurring. The overall probability of a nonvacant use occurring is best seen by visually aggregating the profiles in figure 2, and this implies that as distance increases the probability of vacant land also increases. The other point is that in the vicinity of the CBD, in fact up to 50 units from the CBD, the probabilities sum to greater than 1, that

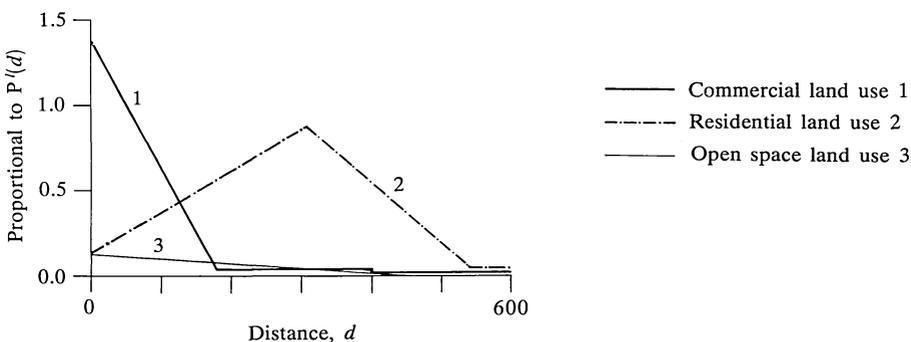


Figure 2. Urban land-use-activity profiles.

is,  $\sum P^l(d) > 1$  ( $l = 1, 2, 3$ ). This does not constitute a problem because the activities are considered in the simulation in order of their economic importance; commercial-industrial are always allocated first, then residential, finally open space. This achieves the following effects.

The probability structure is first set up in the order of importance of these activities. A range of probability is fixed for each activity as follows.  $R^0 = 1$ ,  $R^1 = 1000 P^1(d)$ ,  $R^2 = 1000 [P^1(d) + P^2(d)]$ , and  $R^3 = 1000 [P^1(d) + P^2(d) + P^3(d)]$ . An activity type is allocated by drawing a random number between 1 and 1000 [using RND (1000)]. If the sum of the probabilities is greater than 1, then the commercial-industrial activity will have priority, then residential-housing, finally open space. In fact, when  $d = 0$ ,  $R^1 = 1000 \times 1.38$ , and thus the activity will always be commercial. Only when  $d > 50$  will other activities be 'competing' for allocation. However when  $d > 550$ ,  $R^3 \approx 6$ , and effectively all the activity will be vacant land. In essence, this marks the boundary of the city. These equations thus control many dimensions of urban activity allocation and physical form, and the shape of the city can be quite radically altered by changing the parameters  $\alpha^l$ ,  $\beta^l$ , and  $\lambda^l$ . The values presented were fixed by a process of trial-and-error simulation as well as being judged consistent with simple urban bid-rent and density theory.

#### *Simulation of urban land use*

The fractal simulations involve a straightforward concatenation of the recursive generating process [in figure 1 and equations (1) to (3)] with the general model structure [in figure 2 and applications of equation (4)]. To demonstrate the dependence of pattern and shape on the level of recursion, we have run the model with dimensions similar to those of Greater London (GLC, 1985) for levels of recursion  $0 \leq r \leq 5$ . This produces six simulations which are presented in figure 3 (on coloured pages). These show quite different patterns. Up to level  $r = 2$ , the pictures reveal the coarse triangular mesh used to generate shapes of land-use activity. Moreover, not enough zones are generated to achieve a reasonable distribution of activity types. However for  $r \geq 3$ , the structure becomes much more acceptable. However, by  $r = 5$ , which touches the level of pixel resolution, the pattern looks more like a pointillist painting than a city. The most appropriate-looking images are thus generated for  $r = 3$ , and  $r = 4$ . This is an important point in the simulation of visual realism, and it also suggests that the probability structure of the underlying models is not invariant to scale, an issue which in some senses is obvious, but one which has rarely been explored.

These types of simulation do, however, emphasise the inadequacies of urban models in terms of spatial pattern and visual realism. The images in figure 3 are too compact in that one might expect much greater spread of development as the city expands. Despite the preset wedge-sector geometry, these patterns do not display the classic corridor effects which characterise a radially concentric city. Compare these, for example, especially the images for  $r = 3, 4$ , and  $5$  in figure 3, with the pattern of growth of Greater London as far back as 1944 in Abercrombie's Plan for the metropolis (see *The Planner*, volume 70, number 11, 1984). Thus the immediate advantages of fractal simulation are clear. Spatial effects in models are immediately observable and systematic biases can be detected. Only large-scale simulation can achieve this.

Last, although the dimensions of our simulations are those of London, these simulations are as much London as the famous Mandelbrot-Voss *Planetrise* pictures (Mandelbrot, 1982b, cover and page C9) are the earth viewed from the moon. This is a very important issue in fractal graphics, for in this case, it suggests the sorts of elements required in order to generate minimal city forms.

The whole feel to the images for  $r \geq 3$  is that of a large city like London. In fact, we have cheated slightly, perhaps grossly, by adding the distinctive River Thames to the images after they have been generated. This is a strong perceptual clue to any picture, but even without it the images for  $r \geq 3$  reflect a large concentric city like London. In our fully fledged simulations produced later for Greater London, we will in fact omit the River, for in these later simulations, the shape of the city will be encoded in the input data which reflect the built-up area, and the Greater London County boundary.

### **Explanation and simulation**

#### *The model-building process*

So far the hierarchical structures we have introduced do not relate to any observable characteristics of city systems except in the most superficial way. Clearly for levels  $0 \leq r \leq 2$  in figure 3, the images generated show the strong influence of the triangular hierarchisation and are thus not realistic. When levels with  $r \geq 3$  are reached, the images no longer display the method, in that the concatenation of triangles at these levels produces the sorts of irregularity characteristic of land-use patterns. Thus, in one sense, the triangular subdivision process is scale-dependent. However, in fractal simulation there is still the need for substantive analysis as in other forms of modelling. Indeed, many examples of fractals can only be modelled coherently by defining their intrinsic properties of self-similarity: trees, for example, are self-similar through their mode of reproduction and growth. In geomorphology, the process of weathering and erosion acts in a self-similar fashion. This is clearly true for cities as well and thus hierarchical structure must reflect this.

We can sketch an idealised process of fractal simulation to which we will aspire here and in future research. We begin by identifying hierarchy in the system of interest based on our perception of self-similarity in descriptions, and we are then able to measure whether or not the phenomenon is fractal and whether or not the fractal dimension is invariant to changes in scale. Each stage of measurement and description leads to further development of the underlying process through which the structure can be generated, and this in turn leads to models which are consistent with fractal structure. Once appropriate models, applicable to different levels of the spatial hierarchy, have been developed, other fractal structures utilising such hierarchy and incorporating the application of the underlying models through recursion, can be simulated.

This approach is in fact the classic process of observing a phenomenon, deciding whether it meets any theoretical preconceptions we have, developing a 'best' model structure, and then using this to enable new and different predictions to be made. Essentially this is the process of induction followed by deduction, or in a different sense, analysis followed by synthesis. We can think of induction as a process of building theory from the bottom up, from specifics to universals, whereas deduction is a top-down process in which universals are used to predict specifics. The best expression of this complete process is in the fields of design and problem-solving where problems must be understood (through induction and analysis) prior to their solution (through deduction and synthesis). In fact, in design, methods for analysis and synthesis exist which are based on searching for hierarchical structure: problems are decomposed in the quest to induce their structure and thence composed in the quest to synthesise a solution from the elements (Alexander, 1964; Johnson, 1984). There are parallels with the process used here to enable appropriate description and explanation prior to fractal simulation.

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A simple example which relates to spatial theory is the rank-size distribution of cities. City-size distributions display regular properties which are consistent with subdivision of a national or regional space into market areas whose decreasing size reflects the frequency of spatial dependence and the rarity value of spatial goods. Idealised size distributions can be developed by taking a primate city and its national market area, generating two next-order cities, then four, then eight, and so on; this is the type of method used in central place theory. In terms of our complete cycle of model-building, we first need to identify the hierarchy of market areas, transport routes, population centres, etc, thus explaining spatial structure at different levels. This is accomplished inductively in bottom-up fashion, possibly using clustering-type methods. The simulation then begins from the topmost level in the hierarchy by subdivision and fractal rendering, generating centres and activities at different scales in such a way that lower levels depend on upper. Although there is a sense in which the simultaneity of dependence is treated by correct bottom-up followed by top-down analysis, in terms of fractal simulation which is arbitrarily structured in hierarchical terms the dependence is only one way. In fact, this is a problem with many hierarchical descriptions, for it is clear that any activity at any position in the hierarchy owes its stability to those activities both above and below it. This in fact is the concept of 'niche' and it is something which must be explored in considerable depth in further research on fractal simulation.

#### *Hierarchical models*

In spatial modelling there are some very well-developed techniques to effect this process of hierarchical explanation and simulation. The logical output of a process of continual subdivision is the elemental space which contains the individual, and thus individual behaviour lies at the base of the spatial hierarchy. Such models have been widely developed during the last decade to address problems of discrete choice in the economic domain using standard methods of econometric estimation (Lerman, 1985). These are the models which will be used here, and a particularly attractive feature of them is the fact that they can easily and logically incorporate hierarchical structure: these are the so-called sequential or nested logit models (Hensher and Johnson, 1981).

Very few applications exist as yet of truly spatial discrete choice models and even fewer have been developed in a spatially nested form. Nevertheless, these models appear promising as the basis of the recursive generation of activity through the spatial hierarchy. The other class of models which will be considered at a later stage of this research, and which are related to discrete choice models, are spatial interaction-entropy models. It is well-known that such models have highly articulate properties of spatial decomposition (Roy, 1983) and this also makes them attractive to hierarchical simulation. There are a variety of methods for enabling hierarchy to be defined and built into spatial models, such as the standard multivariate cluster-type techniques as well as methods based on more subjective comparisons such as Saaty's (1980) analytic hierarchy process; these could also prove useful to further research.

In the rest of the paper, we will not attempt to address the full process of hierarchical description through the identification and use of hierarchical models, but we will follow the broad sequence of inductive-deductive stages in the modelling process. We will begin by selecting models for individual choice of housing type and location in Greater London which is the urban region we intend to simulate. This first involves a traditional process of formulating, estimating, and selecting appropriate discrete choice models. Having accomplished this, we will

move on to the simulation in which these discrete choice models are used to predict housing choice at the lowest level of fractal detail generated. In this way, an image of the residential urban structure of Greater London is built up. Hierarchy is still a largely arbitrary affair in this paper, although we will address it in future research. But there are other problems relating to modelling and simulation which emerge and must be dealt with, specifically related to spatial variation.

The logical next step in this work is to develop a 'realistic' version of our hypothetical simulation presented earlier. To this end, we will now sketch the inductive side of this effort, beginning with the theory of discrete choice and its application to housing in Greater London.

### Discrete choice models of urban structure

#### *The standard multinomial logit model*

To set the context, we must review some fairly standard results, but, in doing so, we will adapt discrete choice models to our application and thus only select those aspects which are of relevance here. We will first state the multinomial logit model (MNL) in which we can identify the choice by individuals  $i$ ,  $i = 1, \dots, N$ , of alternative  $k$ , from the set of alternatives  $k = 1, \dots, K$ , where there are  $N$  individuals in the system making choices from  $K$  alternatives. This set of  $K$  is referred to as the choice set and in our applications involves types of housing. The MNL model predicts a probability,  $P_{ik}$ , which is the probability of individual  $i$  choosing house type  $k$  where there are four house types to choose from, and where  $i$  implicitly represents the location of the individual in the city. Thus the model is designed to explain choice in terms of location.

First, we must associate a utility of choosing alternative  $k$  with the individual  $i$ . This utility,  $U_{ik}$ , is usually specified as a linear sum of,  $M$ , exogenous (input) variables which may be specific to the choice in question or nonspecific (generic). In our context, the parameters of these variables are made specific, being referred to as alternative specific constants, but the variables apply to each house type. Then

$$U_{ik} = \sum_m \beta_{km} x_{im} + \varepsilon_{im} \quad m = 1, \dots, M,$$

where the first term on the right hand side of the equation contains strict utility components made up of parameters,  $\beta_{km}$ , and independent variables,  $x_{im}$ , and the error term,  $\varepsilon_{im}$ , reflects differences in tastes, unobservable influences, and such like. The MNL model is derived by assuming that the error components ( $\varepsilon_{im}$ ) are identically and independently distributed, and by maximising utility using the traditional economic logic (Hensher and Johnson, 1981). This random utility derivation of the MNL model is subject to the normalisation

$$\sum_k P_{ik} = 1,$$

and the model is derived as

$$P_{ik} = \frac{\exp U_{ik}}{\sum_l \exp U_{il}} = \frac{\exp \left( \sum_m \beta_{km} x_{im} \right)}{\sum_l \exp \left( \sum_m \beta_{lm} x_{im} \right)}. \quad (5)$$

These sorts of model have been widely applied in transport research, but have also been adapted to a variety of spatial contexts (see Wrigley, 1985). We will not dwell on this, but suffice it to say that equation (5) is a particularly flexible and adaptable model structure.

For purposes of estimation and prediction we need to express equation (5) rather differently. First we must choose one alternative, say  $k$ , as the base or numeraire, and express equation (5) as

$$P_{ik} = \frac{1}{1 + \sum_{l \neq k} \exp \left[ \sum_m (\beta_{lm} - \beta_{km}) x_{im} \right]} \tag{6}$$

We form the ratio of any two probabilities for different choice alternatives using equation (5) and this gives

$$\frac{P_{il}}{P_{ik}} = \frac{\exp \left( \sum_m \beta_{lm} x_{im} \right)}{\exp \left( \sum_m \beta_{km} x_{im} \right)} = \exp \left[ \sum_m (\beta_{lm} - \beta_{km}) x_{im} \right] \tag{7}$$

We can now express  $P_{il}$  in terms of the numeraire  $P_{ik}$  using equations (6) and (7) which simplify to

$$P_{il} = P_{ik} \exp \left[ \sum_m (\beta_{lm} - \beta_{km}) x_{im} \right] \\ = \frac{\exp \left[ \sum_m (\beta_{lm} - \beta_{km}) x_{im} \right]}{1 + \sum_{l \neq k} \exp \left[ \sum_m (\beta_{lm} - \beta_{km}) x_{im} \right]} \tag{8}$$

When  $k = l$ , equation (8) collapses to equation (6). In this paper equation (7) is used in estimation whereas model predictions are made using equation (8).

*Estimation theory*

The logarithm of equation (7) is referred to as the log-odds of alternative  $l$  versus alternative  $k$ , and this is the actual equation which is used in estimation. Then

$$\ln \left( \frac{P_{il}}{P_{ik}} \right) = \sum_m (\beta_{lm} - \beta_{km}) x_{im} = \sum_m \mu_{lm} x_{im} \tag{9}$$

There is a clear interpretation of the parameters in equation (9). If  $\mu_{lm}$  is positive, the choice of alternative  $l$  is more important with respect to the variable  $x_{im}$  than the choice of alternative  $k$ . The reverse is true if  $\mu_{lm}$  is negative, and there is no difference in importance between choices if  $\mu_{lm} = 0$ .

The model parameters in equation (9) are usually estimated using weighted least squares or maximum-likelihood, and here we prefer to use the latter because of the availability to us of Hensher’s BLOGIT computer package (Hensher and Johnson, 1981). To assess goodness of fit we also require the data set of actual choices made which is given as  $F_{ik}$ , where  $F_{ik} = 1$  if individual  $i$  actually chose alternative  $k$ , and  $F_{ik} = 0$ , if this choice was not made. We calibrate the model by maximising the log-likelihood which is given as

$$L(\beta) = \sum_{i,k} F_{ik} \ln P_{ik} \tag{10}$$

and we can also assess the fit as a variation of this likelihood function. A null hypothesis can be set up in which  $\beta_{lm} = 0, \forall l, m$  implying no variation across individuals, that is,  $P_{ik} = P_k, \forall i$ . This can be used to compute the null-likelihood from equation (10) which is given as

$$L(0) = \sum_{i,k} F_{ik} \ln P_k = \sum_k N_k \ln P_k \tag{11}$$

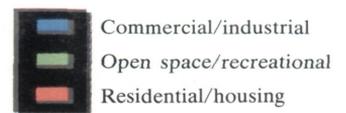
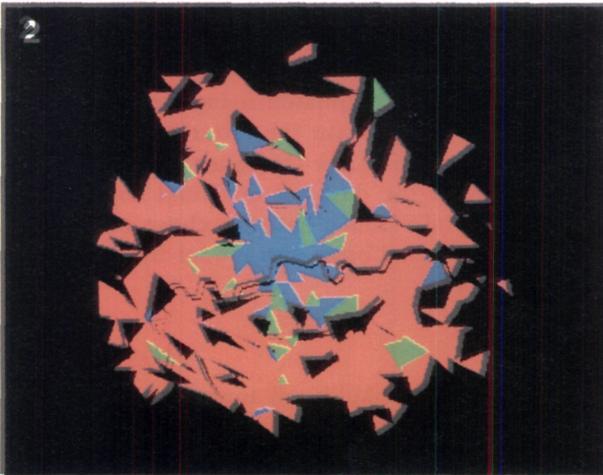
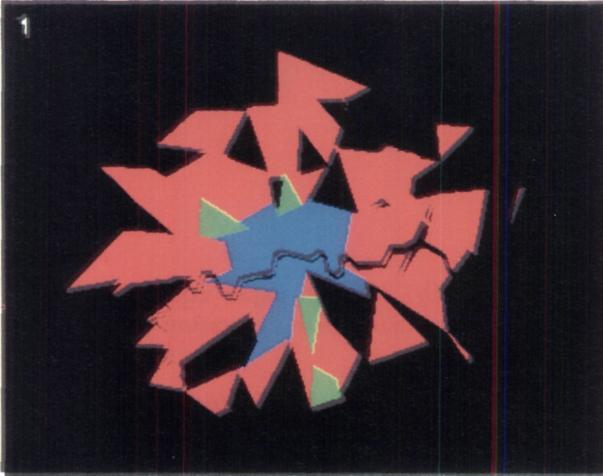
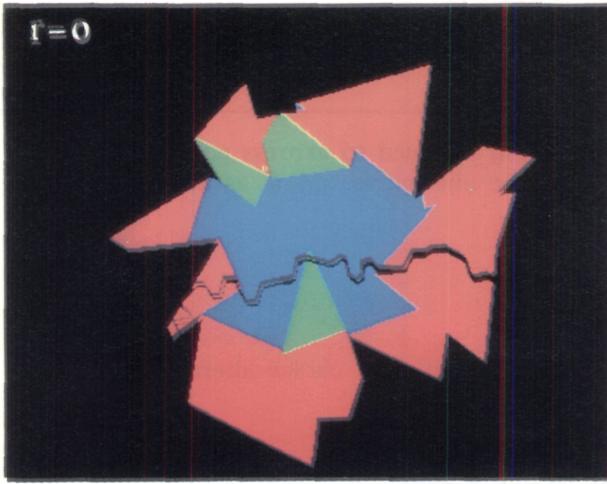


Figure 3. Simulations of land-use-activity structure in a large city at different levels of recursion.

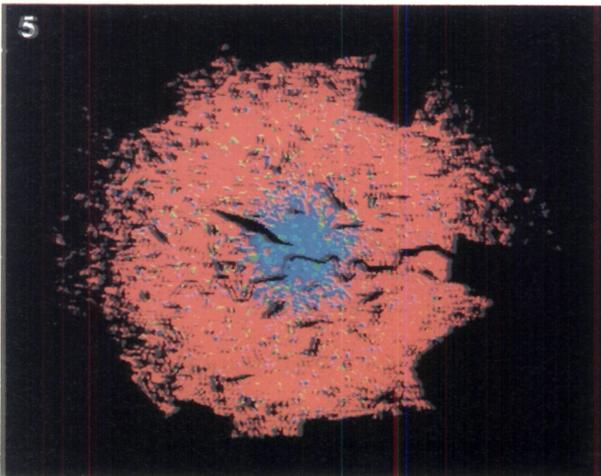
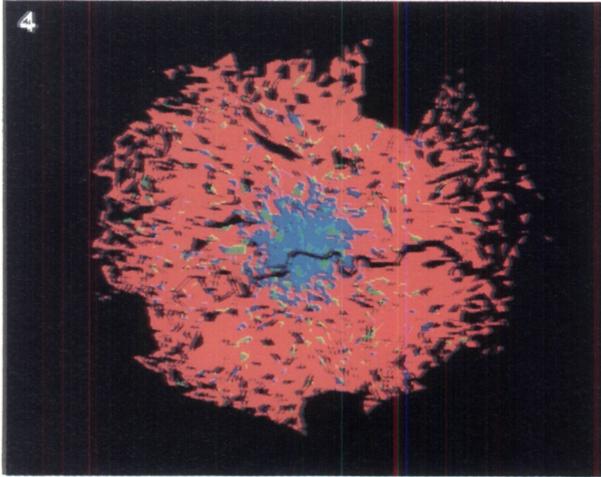
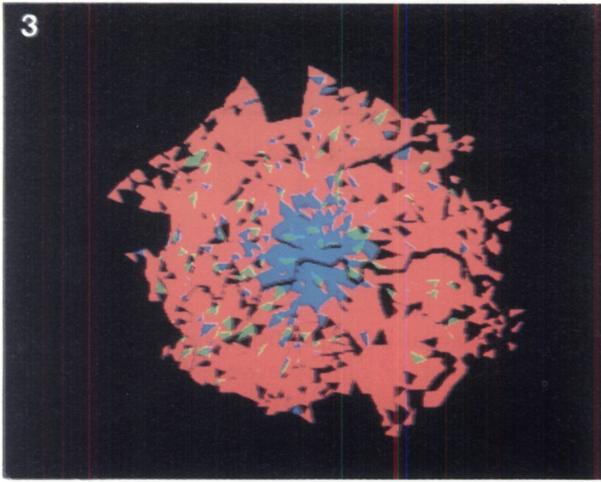


Figure 3 (continued)

where  $N_k$  is the actual number of choices of alternative  $k$  made by all individuals  $i$ . A measure of fit, in some ways similar to the correlation coefficient, is defined as  $\rho^2$ , where

$$\rho^2 = 1 - \frac{L(\beta)}{L(0)}, \tag{12}$$

which varies between 0 and 1. The statistic can also be modified to reflect degrees of freedom, and, typically, good values of  $\rho^2$  range between 0.2 and 0.4. In fact, Hensher and Johnson (1981) argue that any model with  $\rho^2 > 0.2$  is likely to be acceptable. Other measures of fit and diagnostics for log-linear model equations are discussed by Wrigley and Longley (1984), and Wrigley (1985).

*The prediction of model success*

There is a major difficulty in generating less global goodness-of-fit measures for discrete choice models. Because the observed data represent discrete choices  $\{F_{ik}, F_{ik} = 0 \text{ or } 1\}$  whereas the predictions are given as probabilities  $\{P_{ik}, 0 \leq P_{ik} \leq 1\}$ , comparisons at the individual level are meaningless. Thus some aggregation is always necessary. One scheme suggested by McFadden (1979) involves computing expected choices, that is, the numbers of individuals who originally chose alternative  $k$  and are expected to choose alternative  $l$ . In fact, in later simulations we will examine individual predictions but for the applications to London which follow, comparisons between observations and predictions will be confined to success statistics based on expected choices.

To introduce these statistics, first note the structure of the observed choice set  $\{F_{ik}\}$ . Then by definition,

$$\sum_k F_{ik} = 1, \quad \sum_i F_{ik} = N_k, \quad \sum_{i,k} F_{ik} = \sum_k N_k = N. \tag{13}$$

The first equation in (13) implies any individual can only make one choice, the second is the constraint on the number of choices made for each alternative, and the third simply says that the total number of choices made is the same as the number of individuals,  $N$ . The analogous structure for the probability set  $\{P_{ik}\}$  is

$$\sum_k P_{ik} = 1, \quad \sum_i P_{ik} = \hat{N}_k, \quad \sum_{i,k} P_{ik} = \sum_k \hat{N}_k = N. \tag{14}$$

Similar interpretations for equations (14) exist as for those in equation (13), but note that summation of  $\{P_{ik}\}$  with respect to individuals yields predicted numbers of choices,  $\hat{N}_k$ , in contrast to actual numbers,  $N_k$ .

For each individual choice,  $F_{ik}$  (where  $F_{ik} = 1$ ), there is a probability,  $P_{il}$ , that the same individual will make a different choice. The number of such choices across all individuals is the number of individuals who originally chose alternative  $k$  and are expected to choose alternative  $l$ , and this is defined as

$$N_{kl} = \sum_i F_{ik} P_{il}. \tag{15}$$

The set  $\{N_{kl}\}$  is the so-called predicted success matrix. From equations (13) to (15), the matrix has the following properties

$$\sum_l N_{kl} = \sum_i F_{ik} \sum_l P_{il} = N_k, \tag{16}$$

$$\sum_k N_{kl} = \sum_i \left( \sum_k F_{ik} \right) P_{il} = \hat{N}_l. \tag{17}$$

From these definitions it is also clear that

$$\sum_{k,l} N_{kl} = \sum_k N_k = \sum_l \hat{N}_l = N.$$

We can devise a variety of statistics relating to proportions and differences between observed and predicted successes using these aggregations. First we can compute the proportion of correct predictions,  $\eta_k$ , noting that  $N_{kk}$  gives the number of such correct predictions. Then

$$\eta_k = \frac{N_{kk}}{N_k}, \tag{18}$$

which varies between 0 and 1. Total predictive success occurs when  $N_{kk} = N_k, \forall k$ , and  $N_{kl} = 0, k \neq l$ . For the entire system the equivalent statistic to equation (18) is defined as

$$\eta = \sum_k \frac{N_{kk}}{N}. \tag{19}$$

The second index relates to differences between predicted and observed numbers of choices, expressed as proportions or shares. An absolute measure of this index is given by  $N_k - \hat{N}_k$ , and its relative form is defined as

$$\phi_k = \frac{N_k - \hat{N}_k}{N}, \tag{20}$$

which can be positive or negative.

The final index we have computed is called by McFadden (1979) the prediction-success index,  $\sigma_k$ . One problem is that if the predicted choices for alternative  $l$  were much larger than those for the chosen alternative  $k$ , that is, if  $\hat{N}_l \gg N_k$ , then the value  $N_{kl}$  would be affected accordingly. To account for this,  $\sigma_k$  is defined as

$$\sigma_k = \frac{N_{kk}}{\hat{N}_k} - \frac{\hat{N}_k}{N},$$

and an overall index,  $\sigma$ , appropriately weighted, is defined as

$$\sigma = \sum_k \frac{\hat{N}_k}{N} \sigma_k = \sum_k \left[ \frac{N_{kk}}{N} - \left( \frac{\hat{N}_k}{N} \right)^2 \right]. \tag{21}$$

The maximum value of  $\sigma$  occurs when  $\sum_k N_{kk} = N$ , and then

$$\sigma^{\max} = 1 - \sum_k \left( \frac{\hat{N}_k}{N} \right)^2. \tag{22}$$

A normalised measure is given by  $\sigma/\sigma_{\max}$ ; other applications are given by Wrigley (1985). In the empirical work which follows, these indices will be further adapted to aggregations of subsets of individuals located in specific zones; these will be presented below.

### Applications to London

#### *Key variables of urban structure*

Conventional descriptions of urban structure tend to be based on disaggregations of urban activities into land use by type and location. One realisation of conventional structure was used in the demonstration model presented earlier where commercial-industrial (work), residential (living), and open space (leisure) activities were treated in a locational framework which emphasised in diverse ways the radial and concentric nature of the contemporary city. It is not possible to

take this model further to the applications stage here, largely because we do not have easy access to a comprehensive land-use-activity database. Moreover, we are interested in developing more formally structured discrete choice models which can be embedded within the fractal simulation, thus enabling us to assess the impact of individual spatial choice behaviour in the larger picture.

Another consideration which has guided us is not just the absence but the availability of data. We have access to a large-scale housing survey—the English House Condition Survey (EHCS: DoE, 1978; 1979)—which was conducted in 1976. This was based on a fairly low sample of households in England, something in the order of 1 in 3000, but this represents an easily available, highly disaggregate data source and thus we have chosen to make use of it. One of us (Longley), has already had experience in calibrating logit models of housing-tenure choice using this data set, and the experience gained has been invaluable in orientating certain aspects of this project (Longley, 1984).

We have chosen housing type as the key variable defining urban structure which is a major category in the EHCS data. Houses are classified into five types: purpose-built flats, converted flats, terraced houses, detached or semidetached houses, and a miscellaneous group. House type is a particularly clear way of representing urban structure, for different areas of the city are often perceived generally in terms of house type; historically, cities have grown reflecting different house types, and house type seems to relate to how far people wish to live from the CBD. Cities are often articulated as spatial patterns, with flats near the centre, terraced houses occupying the inner suburbs, detached or semidetached the outer suburbs, each ring reflecting a stage in city growth. Thus density and distance variables are indirectly reflected in house type and in the case of London, this is particularly relevant in that the city is strongly monocentric, has a well-developed market for flats, and has been economically buoyant for several centuries. In our applications, we have in fact excluded the miscellaneous category because it acts as a residual category and contained less than 2% of the observations available in the database.

Choice of house type lies at the base of several contemporary theories of urban structure which integrate two important constructs. First, in bid-rent theory an implicit trade-off in housing decisions is postulated between housing space and type versus proximity or distance to central urban functions; and urban growth and dynamics (as manifest by filtering, suburbanisation, urban renewal, etc, and as expressed in the age of the stock) exhibit an identifiable correspondence with distinctive dwelling types such as subdivided central-city houses, suburban semidetached homes, purpose-built flats in revitalised inner-city neighbourhoods, and so on. The implication is that dwelling and neighbourhood type are clearly related to distance from the CBD and the date at which the land parcel was integrated (or reintegrated) into the contemporary urban development process.

Thus age and distance represent key determinants of urban structure. In designing the models, it was thought important to keep the variables in the models as simple as possible and, at the same time, easily measurable. We also considered neighbourhood quality at an early stage, but eventually dropped this to keep the model simple; in any case, neighbourhood quality was subjectively specified in the EHCS data and thus difficult to predict generally. Age of house in which the household respondent resided was available in the survey, but distance from the CBD was not, and this constitutes a problem. Each individual was not coded by exact location in the data set but located by borough of which there are thirty three in Greater London. What we have done in measuring distance is to locate a centroid in each borough and use airline distance from this to a point in the City of London.

Another consideration involved the fact that when we embed the discrete choice models into the large-scale (fractal) simulations, we require data on age of housing and distance to the CBD at every conceivable point of residential development in Greater London. These data are amongst the easiest to obtain from independent sources. We used an age distribution for housing measured over seven levels available from the Greater London Council (GLC) Intelligence Unit Library. Distance is measurable directly from the map and neighbourhood quality, although available from the GLC, did not appear to match that used in the EHCS and was thus excluded at an early stage of model estimation.

The general form of the models we have estimated, in log-odds form, is

$$\ln \frac{P_{il}}{P_{i1}} = \mu_{l0} + \mu_{l1} D_n + \mu_{l2} A_i \quad i \in Z_n, \quad l = 2, 3, 4, \tag{23}$$

where  $Z_n$  is the spatial definition of the borough  $n$ . The log-odds equation is normalised with respect to the probability of choosing a purpose-built flat,  $P_{i1}$ , and the other choices involve converted flats ( $l = 2$ ), terraced houses ( $l = 3$ ), and detached or semidetached homes ( $l = 4$ ).  $A_i$  is the age of the dwelling in which individual  $i$  resides, and  $D_n$  is the distance from the CBD to the centroid of the borough in which individual  $i$  resides.

In essence, we assume that  $D_i$  is unobserved and that equation (23) is an appropriate approximation to the underlying discrete choice model analogous to equation (23) in which  $D_i$  replaces  $D_n$ . Equation (23) will only be acceptable if  $D_n$  is the mean distance, and the sum of the differences around  $D_n$  in the borough cancel. Formally, if  $D_i = D_n + \varepsilon_i$ , where  $\varepsilon_i$  is the ‘error’ difference between the mean and the actual distance to individual  $i$ , the average  $D_n$  can be defined in terms of  $D_i$  as

$$\sum_{i \in Z_n} \frac{D_i}{N_n} = D_n + \sum_{i \in Z_n} \frac{\varepsilon_i}{N_n}, \tag{24}$$

where  $N_n$  the number of individuals in  $Z_n$ . Quite clearly, the mean will only be equal to  $D_n$  if  $\sum \varepsilon_i = 0$  ( $i \in Z_n$ ), that is, if the errors around the mean are self-cancelling in total. We cannot explore the detailed implications of this aggregation, but it is important to further research. Discrete choice theory is strangely deficient in clear discussion of the spatial aggregation problem, with the exception of important work by Anas (1981; 1982).

Before we broach questions of model selection and estimation, we will sketch how the model we are working with could be developed in nested fashion, to account not only for the aggregate form of the distance data but also for more substantive questions related to the sequence of spatial decisionmaking. Because distance to CBD is only available at borough level, it might make sense to conceive the house-type-residential-location process as one in which a choice of neighbourhood type is made first on the borough ( $Z_n$ ) level, in terms of neighbourhood quality and distance from the CBD, and then the choice of house type made at the individual location with respect to age. Such a model could be written as

$$P_{iqk} = P_{iq} P_{ik|q},$$

where  $P_{iqk}$  is the probability of an individual  $i$  choosing neighbourhood type  $q$  and house type  $k$ ,  $P_{iq}$  is the probability that the individual chooses neighbourhood type  $q$  at borough level, and  $P_{ik|q}$  is the probability the same individual then chooses house type  $k$ , having chosen neighbourhood type  $q$ . Such a sequence could be structured so that the fractal simulation enabled neighbourhood type to be chosen at an appropriate level of fractal resolution, and house type at a lower level.

Although neighbourhood type is predicted here, this could be suppressed if it were regarded as only an intermediate variable of little visual significance. There are many issues to resolve here, but some work along these lines in an industrial location context by Hayashi and Isobe (1985) looks promising, as does the theoretical work of Roy (1983). Nested models of this type will be pursued in further research.

*Model selection and estimation*

We developed a number of preliminary specifications of the model before we decided upon equation (23). We first estimated some models based on housing tenure, but then dropped these in favour of house type when our ideas relating to urban structure became clearer. We began with five categories of house type including miscellaneous, but dropped this when it appeared nonsignificant in explanation. We then estimated the house-type model with all combinations of up to three exogenous variables: age and distance which we eventually selected, but also neighbourhood quality. With three variables, there are seven models which can be specified, and the global fit of each of these seven is given in table 1.

By far the best of the models are the two which included the age and distance variables. These models are the only ones which reach the threshold of acceptability in which  $\rho^2 > 0.2$ , suggested by Hensher and Johnson (1981). The best model also includes neighbourhood quality, but the percentage increase in fit, between the model without this variable and that with, is less than 5% and thus neighbourhood quality has been omitted. Other reasons relate to the fact that neighbourhood quality is difficult to produce in a consistent and comprehensive database for London, and to the fact that we have severe memory problems in our fractal simulations which mean we need to hold both input and output data in screen memory simultaneously. This limits the number of variables we can deal with, and thus neighbourhood quality was felt to be dispensable.

We will now examine the discrete choice model estimated for the age–distance variables in equation (23). The three fitted equations are given as follows, where for purpose-built flats  $l = 1$ , for converted flats,  $l = 2$ , for terraced houses  $l = 3$ , and for detached or semidetached houses  $l = 4$ ,

$$\ln \left( \frac{P_{i2}}{P_{i1}} \right) = -4.862 + 0.034 D_n + 0.067 A_i , \tag{25a}$$

$\begin{matrix} [-7.984]^* & [0.754] & [10.599]^* \\ (0.609) & (0.045) & (0.006) \end{matrix}$

$$\ln \left( \frac{P_{i3}}{P_{i1}} \right) = -3.605 + 0.177 D_n + 0.052 A_i , \tag{25b}$$

$\begin{matrix} [-9.818]^* & [6.735]^* & [11.439]^* \\ (0.367) & (0.026) & (0.005) \end{matrix}$

**Table 1.** Global fits of models incorporating age, distance, and neighbourhood quality.

Independent variables	$\rho^2$
Age	0.118
Distance	0.089
Neighbourhood quality	0.069
Age and distance	0.207 <sup>a</sup>
Age and neighbourhood quality	0.123
Distance and neighbourhood quality	0.095
Age, distance, and neighbourhood quality	0.218 <sup>a</sup>

<sup>a</sup> Acceptable models within the Hensher–Johnson limit,  $\rho^2 > 0.2$ .

and

$$\ln \left( \frac{P_{i4}}{P_{i1}} \right) = \begin{matrix} -5.737 & +0.354 & D_n & +0.046 & A_i, \\ [-12.143]^* & [11.379]^* & & [9.102]^* & \\ (0.472) & (0.031) & & (0.005) & \end{matrix} \tag{25c}$$

$$\rho^2 = 0.207, \quad N = 809,$$

where *t*-statistics are given in square brackets, standard errors are given in round brackets, and \* denotes a significant *t*-statistic.

Note that the log-odds is essentially the log-likelihood that individual *i* will select the numerator alternative rather than the denominator alternative. In view of the aggregated nature of the distance data, the  $\rho^2$  of 0.207 indicates a reasonable degree of overall fit, and the variable parameters and their corresponding *t*-statistics lend support to our a priori expectations. Equations (25b) and (25c) imply that terraced and detached or semidetached houses are both likely to be further from the CBD and to be older than purpose-built flats; and equation (25a) suggests that converted flats are likely to be older than their purpose-built counterparts.

These interpretations can only be borne out by a full-scale simulation and the pattern of coefficients suggests that flats of both kinds are nearest to the CBD, and terraced, then detached or semidetached houses are further away, if we assume that terraced houses are older than detached or semidetached. As we intend the simulation to be entirely spatial, and spatial structure is not apparent from the model fits presented so far, we need to see how well the models perform spatially at an aggregate level. The obvious level on which to perform such spatial analysis is the borough, for it is at this level that all the model variables are similarly aggregate. We will present our analysis visually in the next section where the predictive success indices of the models are mapped for the thirty-three boroughs.

*Model performance and reestimation*

We have already shown that it is necessary to aggregate individual predicted probabilities so that we can enable some comparison with the observed data. To this end, we introduced McFadden's (1979) predicted success matrix in equations (13)–(17), and then presented various indices of success in which correct proportions, and differences between observed and predicted choices were computed in equations (18)–(22). However, it is possible to compute equation (15), the numbers of persons originally choosing house-type *k* and predicted to choose house-type *l*, for subsets of individuals, in particular individuals residing in certain zones, in this case boroughs,  $Z_n$ . In all the indices which follow,  $N_{kl}$  is replaced with

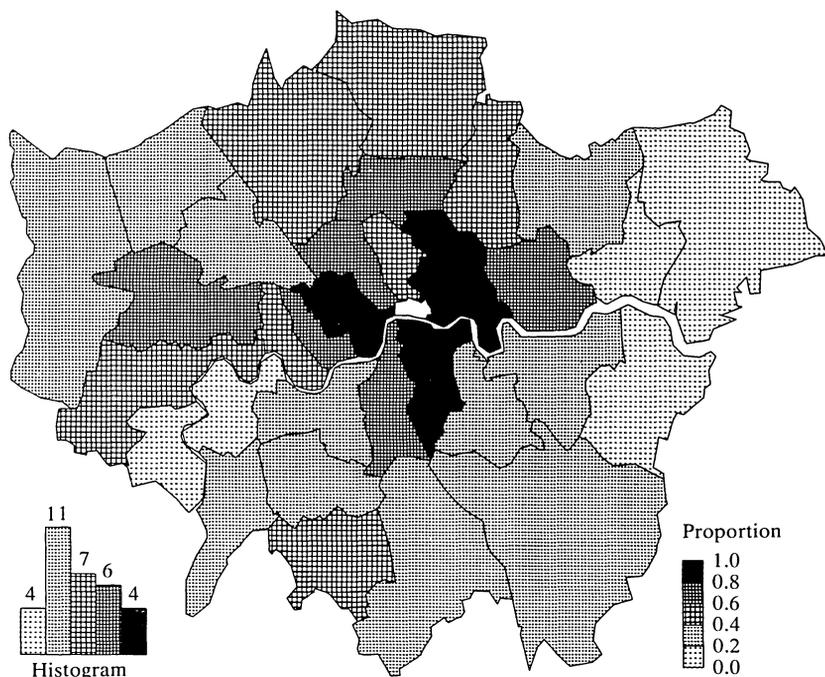
$$N_{kln} = \sum_{i \in Z_n} F_{ik} P_{il}, \tag{26}$$

where  $N_{kln}$  is the number of individuals originally choosing house-type *k* and predicted to choose house-type *l* in borough  $Z_n$ .

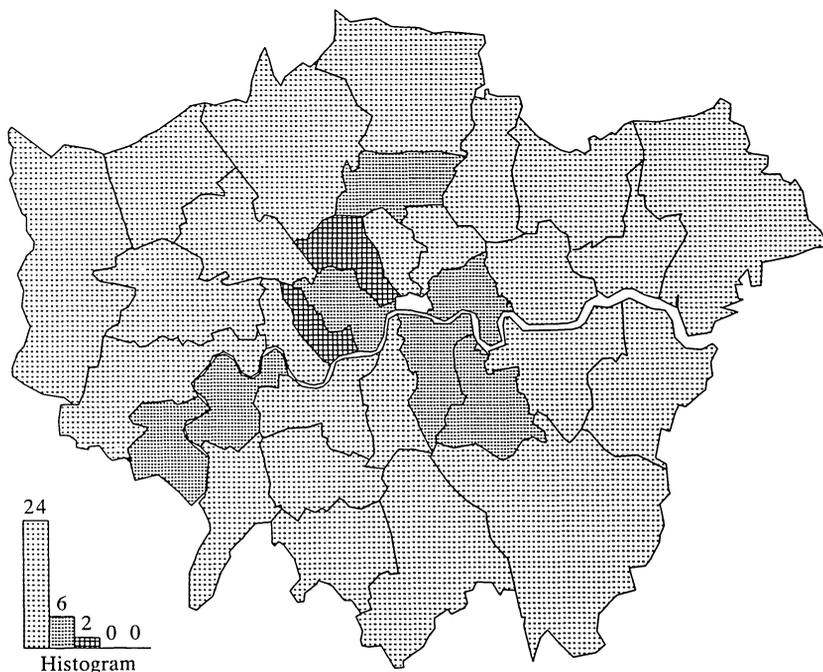
The proportion of correct predictions defined in equations (18) and (19) for the whole of Greater London can act as a basis for comparison with their zonal equivalents. These statistics were computed using the model in equations (25) as

$$\eta_1 = 0.533, \quad \eta_2 = 0.198, \quad \eta_3 = 0.433, \quad \eta_4 = 0.397 .$$

These indices seem rather low; only in the case of purpose-built flats is there a better than 50% success rate, and converted flats are poorly predicted. The overall percentage of correct predictions from equation (19) is computed as  $\eta = 0.432$ , which is an appropriate average of  $\{\eta_k\}$ . The spatial (zonal  $Z_n$ ) equivalents of  $\eta_k$ , called  $\eta_{kn}$ , are mapped across the thirty-three boroughs in figure 4. (Note that in

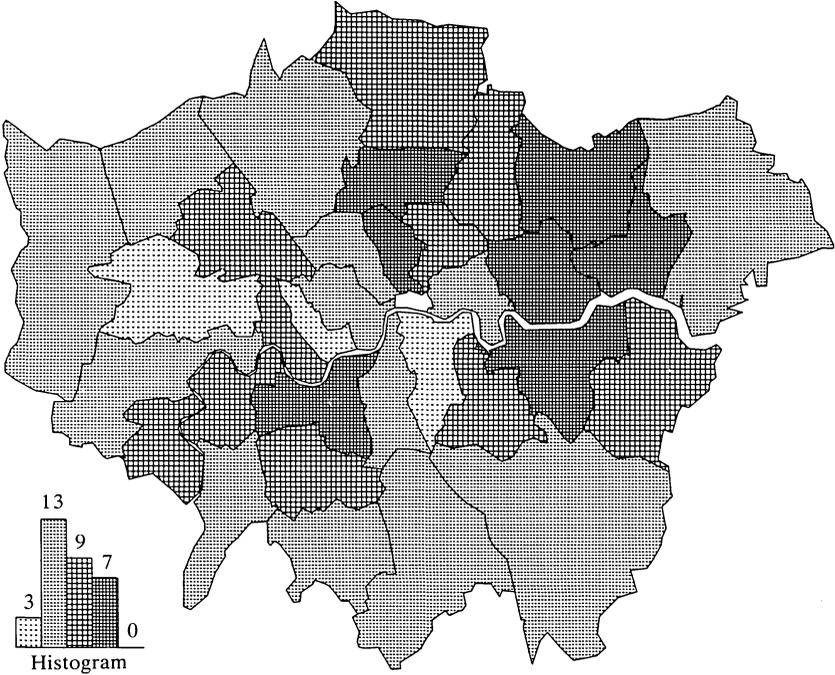


(a)

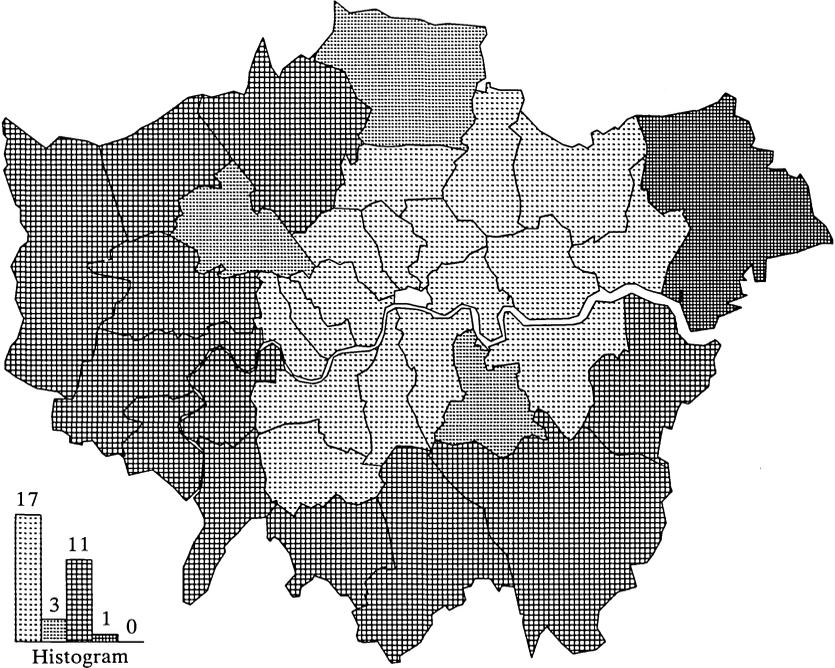


(b)

Figure 4. Proportions of correct choices of house type: (a) purpose-built flats, (b) converted flats, (c) terraced houses, (d) detached or semidetached houses.

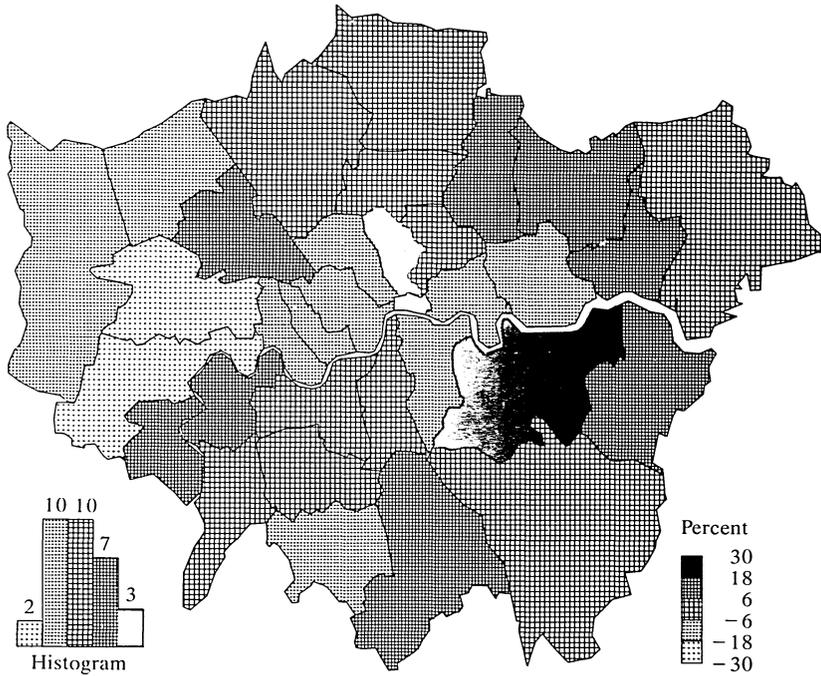


(c)

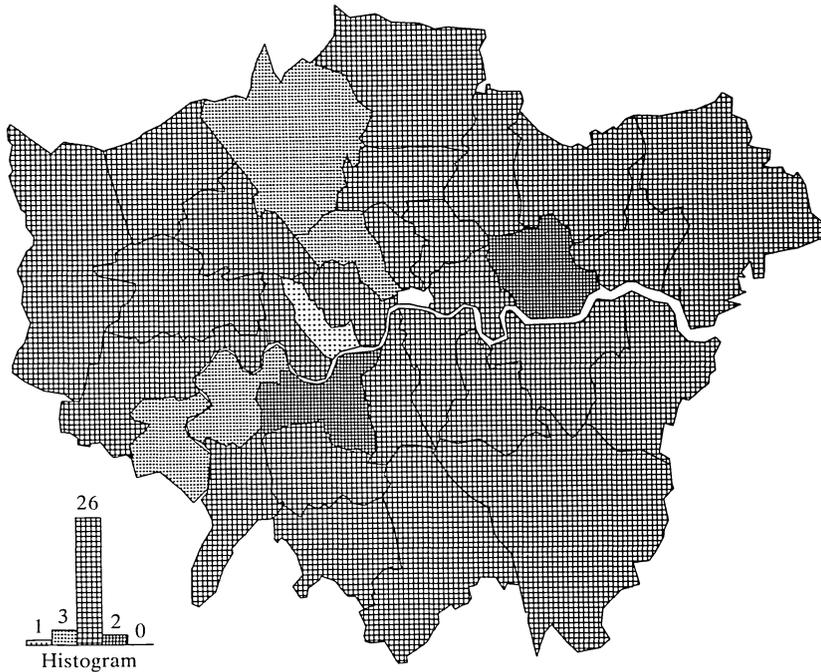


(d)

Figure 4 (continued)

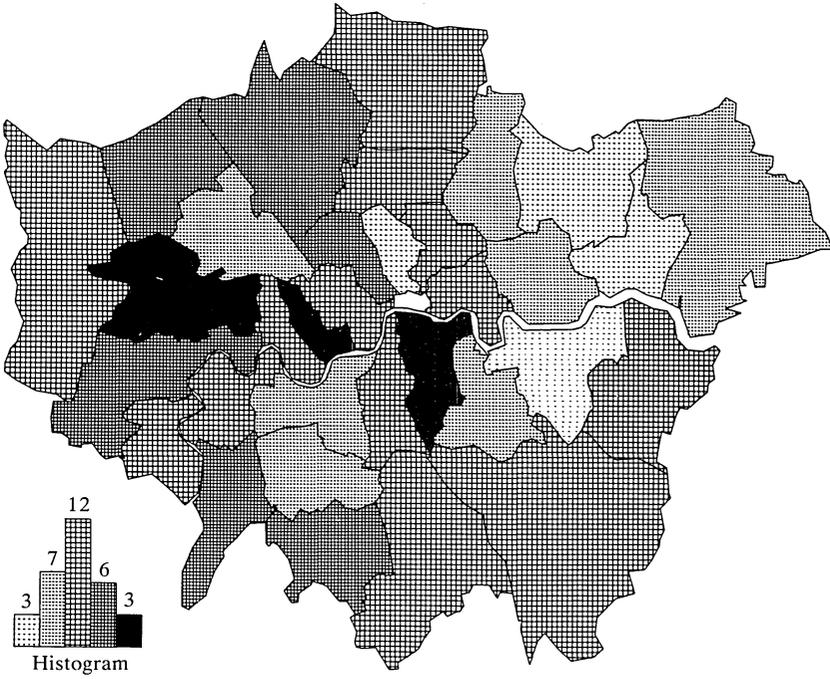


(a)

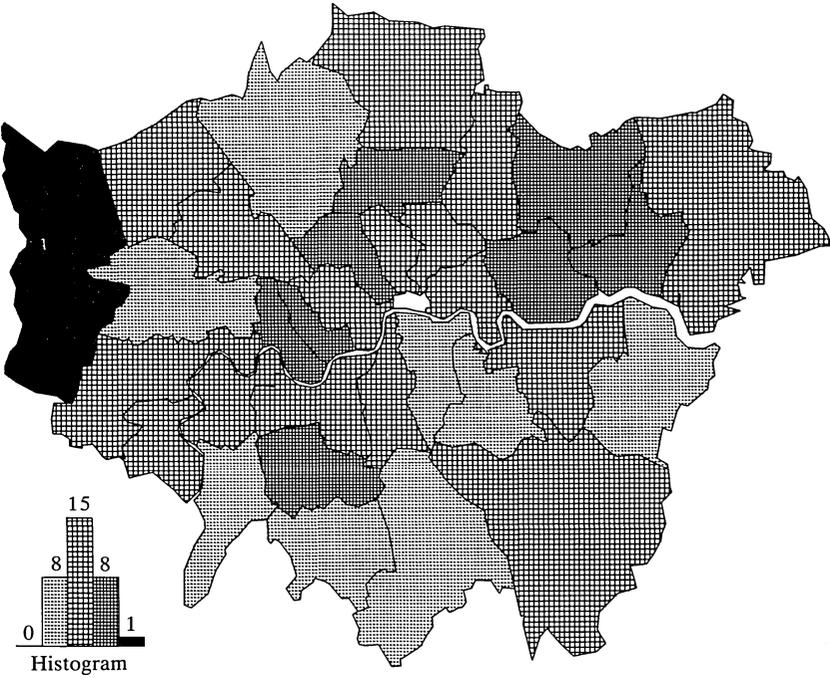


(b)

**Figure 5.** Differences in observed and predicted housing choices: (a) purpose-built flats, (b) converted flats, (c) terraced houses, (d) detached or semidetached houses.



(c)



(d)

Figure 5 (continued)

all these types of map, the City of London Borough does not contain any observations and thus is not shaded). These percentage correct predictions show a much wider range of variation. In general, purpose-built flats are better predicted closer to the CBD, whereas the reverse holds for detached or semidetached houses. The distribution of converted flats generally shows a low percentage correct prediction, with a slight increase towards the CBD, whereas terraced houses show a less distinctive spatial pattern with a slight increase in performance towards the periphery. In fact, figure 4 contains the clearest demonstration we have that individual choice behaviour varies spatially. The obvious conclusion is that there are two sets of models, one for inner, the other for outer London, but before we consider these further, we will examine other indices of predictive success.

Indices of the percentage difference between observed and predicted choices given by equation (20)  $\{\phi_k\}$  have been computed in spatial equivalent form and are mapped in figure 5. The patterns are much less clear than those in figure 4. For purpose-built flats, the largest differences are in the inner suburbs, and the smallest in the centre and the west. For converted flats the pattern is much more random, with a slight bias towards higher differences in the inner suburbs. For terraced houses, the inner suburbs show higher levels of under- and overprediction of shares, whereas the outer suburbs display the greatest differences in the case of detached or semidetached houses. These maps are more difficult to interpret than their counterparts in figure 4. What they do show, however, is that there are both sectoral and concentric-geometric spatial biases in the pattern of predictions, which can only be accounted for by the addition of new and different explanatory variables (and the possible deletion of one of the existing ones), or by the development of models which accept these spatial differences. We will pursue the latter course.

To conclude, it is useful to examine the pattern of overall correct predictions from equation (19), computed and mapped spatially, and this is presented in figure 6. The best predictions are recorded in and near the centre and in the outermost suburbs. This suggests the need for two separate models of individual choice behaviour, one for inner zones, the other for outer zones. The need for this distinction is even clearer when the normalised success index computed from the spatial equivalents of equations (21) and (22), and defined as  $\sigma_n/\sigma_n^{\max}$ , is examined. This is mapped in figure 7, and shows that the best predictions occur nearest the CBD, the worst in the far western and eastern suburbs. On this basis, we decided to reestimate our models based on equation (23) for inner and outer London, where inner London is based on the thirteen boroughs which constitute the Inner London Education Authority (ILEA).

The sample size of 809 observations was divided into 337 based on the inner boroughs, the remaining 472 comprising the outer boroughs. First equation (23) for the inner boroughs was estimated as

$$\ln \left( \frac{P_{i2}}{P_{i1}} \right) = -5.446 + 0.194 D_n + 0.061 A_i, \tag{27a}$$

$$\begin{matrix} [-5.849]^* & [1.478] & [8.305]^* \\ (0.931) & (0.131) & (0.007) \end{matrix}$$

$$\ln \left( \frac{P_{i3}}{P_{i1}} \right) = -5.106 + 0.430 D_n + 0.050 A_i, \tag{27b}$$

$$\begin{matrix} [-7.046]^* & [4.106]^* & [8.722]^* \\ (0.725) & (0.105) & (0.005) \end{matrix}$$

$$\ln \left( \frac{P_{i4}}{P_{i1}} \right) = -7.430 + 0.546 D_n + 0.050 A_i, \tag{27c}$$

$$\begin{matrix} [-5.810]^* & [3.203]^* & [5.767]^* \\ (1.279) & (0.171) & (0.009) \end{matrix}$$

$$\rho^2 = 0.228, \quad N_{\text{inner}} = 337,$$

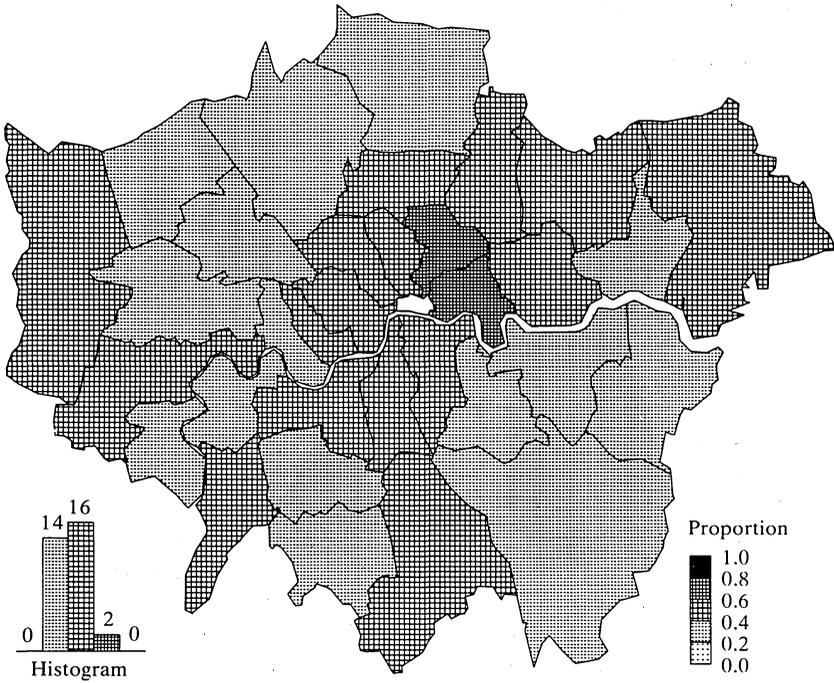


Figure 6. Proportions of correct choices of house types.

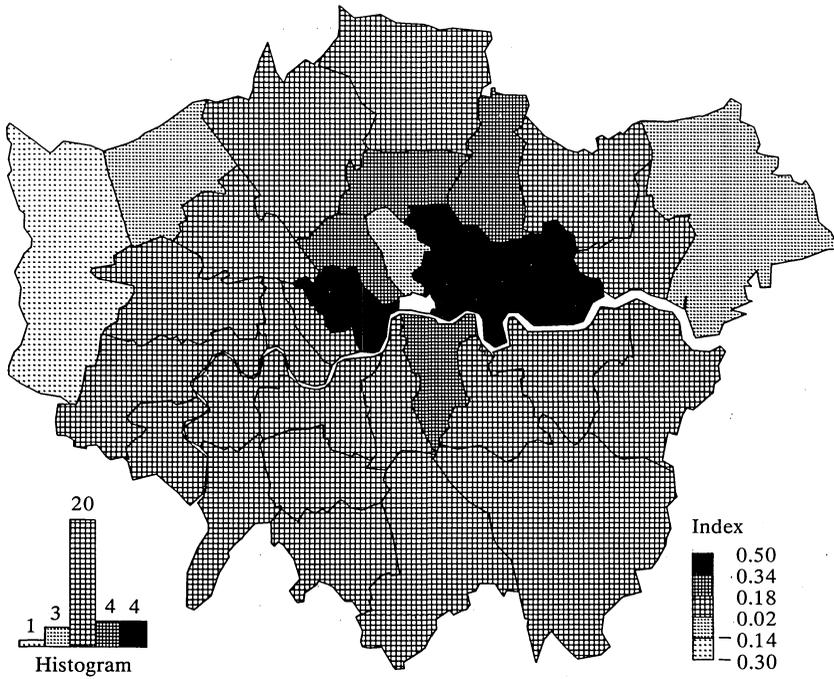


Figure 7. Overall normalised success indices,  $\sigma_n / \sigma_n^{\max}$ , for all house types.

and the appropriate equation for the outer boroughs was estimated as

$$\ln \left( \frac{P_{i2}}{P_{i1}} \right) = -5.119 + 0.030 D_n + 0.075 A_i, \quad (28a)$$

$\begin{matrix} [-3.307]^* & [0.280] & [6.514]^* \\ (1.548) & (0.106) & (0.012) \end{matrix}$

$$\ln \left( \frac{P_{i3}}{P_{i1}} \right) = -1.300 - 0.009 D_n + 0.059 A_i, \quad (28b)$$

$\begin{matrix} [-1.754] & [0.170] & [7.450]^* \\ (0.741) & (-0.051) & (0.008) \end{matrix}$

$$\ln \left( \frac{P_{i4}}{P_{i1}} \right) = -3.537 + 0.186 D_n + 0.051 A_i, \quad (28c)$$

$\begin{matrix} [-4.740]^* & [3.816]^* & [6.538]^* \\ (0.746) & (0.049) & (0.008) \end{matrix}$

$$\rho^2 = 0.106, \quad N_{\text{outer}} = 472.$$

What becomes apparent in terms of the  $t$ - and  $\rho^2$ -statistics is that the inner London model [equations (27)] performs very much as the original model [equations (25)] whereas the outer London results are rather different. Equation (28) reflects a diminished role of the distance variable (two of its associated  $t$ -statistics are insignificant, and one parameter exhibits an unexpected sign) which contributes towards a much lower  $\rho^2$  goodness-of-fit measure. We might rationalise this in terms of our previous land-use theory as follows: although difficulties of accessibility constrained the physical growth of London up until World War 1, the subsequent innovation of mass transit and the automobile rapidly opened up large tracts of land for development.

Because most of this development occurred over very large areas, the form of physical development is much less likely to exhibit a very close and identifiable correspondence with distance from the CBD. Reestimation of our model for outer London without the distance variable yields

$$\ln \left( \frac{P_{i2}}{P_{i1}} \right) = -4.695 + 0.075 A_i, \quad (29a)$$

$\begin{matrix} [-7.388]^* & [6.490]^* \\ (0.635) & (0.011) \end{matrix}$

$$\ln \left( \frac{P_{i3}}{P_{i1}} \right) = -1.399 + 0.059 A_i, \quad (29b)$$

$\begin{matrix} [-4.990]^* & [7.470]^* \\ (0.280) & (0.008) \end{matrix}$

$$\ln \left( \frac{P_{i4}}{P_{i1}} \right) = -0.881 + 0.048 A_i, \quad (29c)$$

$\begin{matrix} [-3.347]^* & [6.241]^* \\ (0.263) & (0.008) \end{matrix}$

$$\rho^2 = 0.078, \quad N_{\text{outer}} = 472,$$

The  $\rho^2$ -statistic is less than that for equations (28) and thus the model has not been used in the simulations which follow. At this point, we can conclude our section on estimation. Many avenues remain unexplored, but several models have been tested and we will take forward those in equations (25), (27), and (28).

### Fractal simulation of house type and location

#### *The graphics database*

One of the more obscure reasons for developing such a simplified model based on age and distance can now be made clear. Age is a spatially extensive variable, whereas distance is a property of space itself. Thus it is possible to display a single map shaded according to age from which distance can also be read, in

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particular, distance to some fixed point from any other. If we had more than a single extensive variable, age *and* neighbourhood quality say, these could not be represented on the same map in easily codable form. Clearly, it is useful for ease of interpretation to have a single map of input data, for this can be directly associated with a map of the outputs from the models.

In fact, the need to store data in map form is essential, for the fractal simulation operates on a graphics microcomputer in which only 8K of memory is available for program and data, 20K of memory being given over to the graphics screen. Although it could be possible to store data on disc, and thus include a larger number of independent spatial variables, the continual reading and writing required would make the operation of the model prohibitively slow. In fact, because the data are spatially extensive, it is essential to store them in screen mode, for the resolution we are working with involves  $160 \times 256$  pixel points which makes any form other than screen storage extremely problematic. The data on age are thus stored as a screen map, and airline distance is easy to compute as a function of screen coordinates which in turn are a function of the screen addressing.

The age data were made available by the GLC Intelligence Unit in seven age groups which were coded in grid fashion, and coloured in the screen memory according to the age group. The screen map is shown in figure 8 (on the coloured pages). The following average ages in years define the seven ranges in question:  $\rightarrow 8 \rightarrow 26 \rightarrow 48 \rightarrow 78 \rightarrow 110 \rightarrow 150 \rightarrow 175 \rightarrow$ . These represent weighted averages which reflect the distribution of housing in any grid square. Distance from the CBD to borough centroids is measured in kilometres, the GLC boundary being about 24 km maximum from the City and the ILEA boundary used for the inner London model being about 13 km distant. Note also that the shape of urban development in London is coded into the data through grid squares coloured on a black background which does not contain housing. These represent 'vacant' land in the sense used earlier, although in these applications the model in no way predicts this.

The way the simulation works involves first loading the age map into the screen memory from file. Then the fractal simulation begins in the order used previously in the demonstration model, and when the appropriate level of fractal detail is reached, the program retrieves the colour of the centroid of the triangle space reached from the screen, converts this into an age value, computes distance, and uses these variables in the model structure based on equation (23) to compute the probability of house type. Thus the simulation works by replacing the regular gridded age map by the irregular fractal land-use pattern in an almost literal sense. This is a rather innovative technique, for input is immediately converted to output and this occurs directly 'before your eyes'. In a sense, it is a version of the WYSIWYG principle ('what you see is what you get') which is central to many operations with graphics computers. A note on technical detail is required. The machine in question is a BBC Micro, the simulation operating in mode 2 (screen resolution  $160 \times 256$  pixels) with sixteen colours. Eight colours are reserved for the age map (seven ages and one vacant land use) and five are used in choosing house type (four types and one vacant land use). The process of replacement is not as clear as it might be because only eight absolute colours are available, hence the replacement of the input map with the output map uses similar colours and is only distinguished in terms of its irregularity. A quick idea of the simulation is achieved if it is run with recursive level  $r = 0$ , or  $r = 1$ .

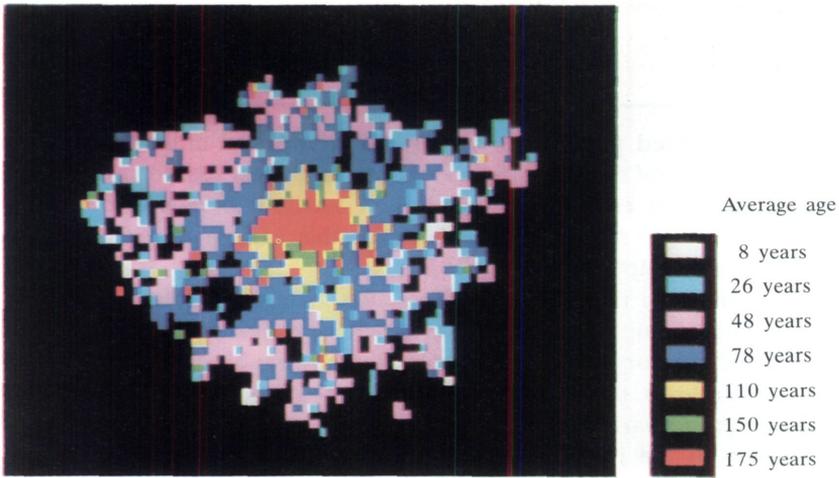


Figure 8. The graphical database: age of housing in London.

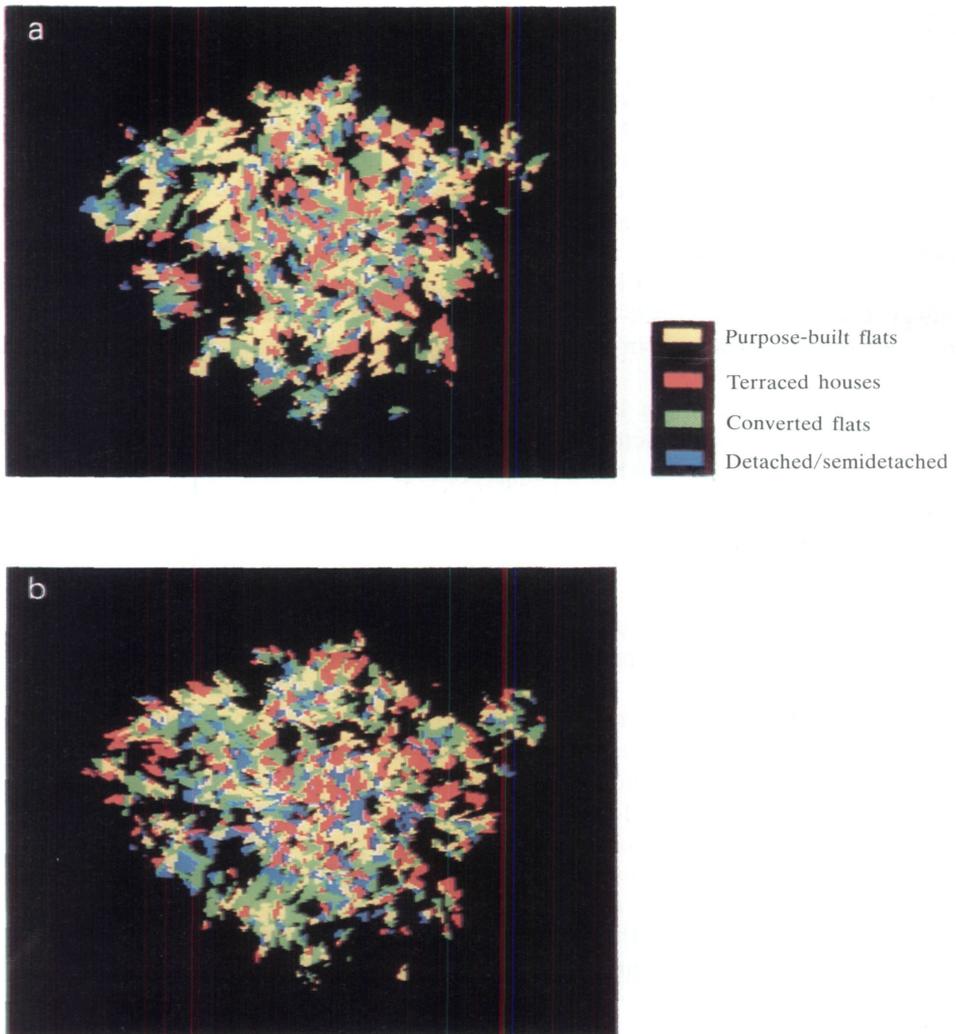


Figure 9. Random simulations of house type and location in London.

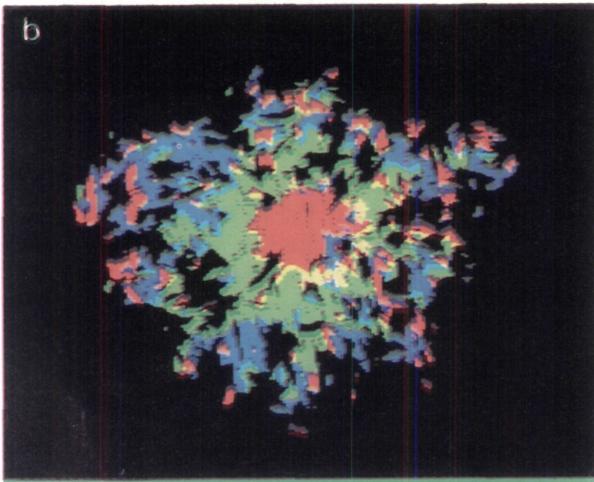
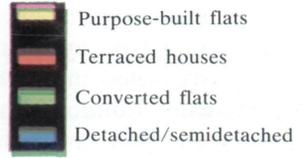
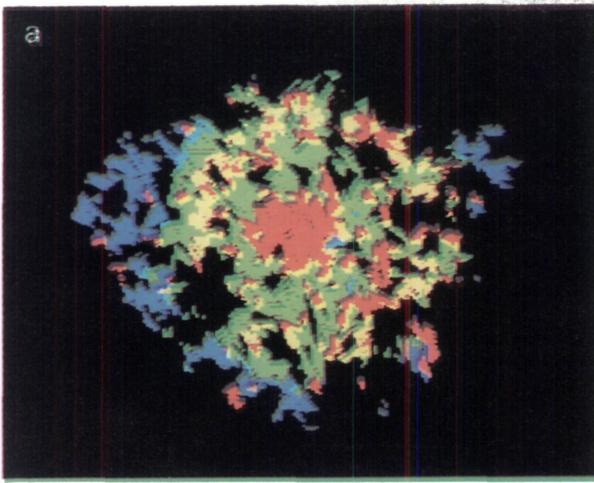


Figure 10. Deterministic simulations of house type and location in London.

### *Random and deterministic simulations*

The process of fractal simulation is essentially the same as that used previously in the demonstration. The only difference relates to the way the input data are stored and sampled and the way the probability models are developed. Four land-use types based on housing, rather than three based on activities, now form the simulated urban structure. The area over which the simulation is operated is fixed and, in a sense, residential location is already predetermined through the data, and thus it is only house type by location which varies.

We have already noted that two model structures are to be used: that based on the whole of Greater London, which uses equations (25), and that based on the distinction between inner and outer London, which uses equations (27) and (28). In these simulations, we work at recursive level  $r = 4$  which essentially fixes fractal detail at just above the pixel level of the screen. Each simulation takes about 3 hours and involves examining  $10 \times 4^{r+1} = 10240$  randomly positioned contiguous triangles which form the network of fractal detail at the lowest level of resolution. In fact, the models are based on 809 data points, and in the area in question there are in excess of 3 million households, thus the simulation itself is still very much in the nature of a sample-style exercise in which an 'average' individual residing at the lowest level of fractal detail makes a house-type choice which is then assumed to be typical of all individuals at that level and in the space which contains that location.

The other issue involves the conversion of probabilities  $\{P_{ik}\}$  into discrete choices. In the demonstration model, a random simulation was adopted in which choice of land use was accomplished according to the probability range fixed by the land-use models but ultimately determined using a random number device. The resultant outputs were very satisfactory because the probability profiles were quite distinct, thus enabling fairly clear decisions to be made and characteristic spatial patterns to emerge. Here, however, the probability profiles of the house-type models are much less different from one another, and thus to develop clearer spatial patterns, we have also introduced a deterministic simulation. This simulation is based on choosing a house type according to the rule

$$\text{type} \leftarrow \max_k \{P_{ik}\}, \quad (30)$$

which simply makes the choice according to that alternative which has the maximum probability for individual  $i$ .

We can now show the simulations. We first present the random simulations which are based on equations (25), then equations (27) and (28). These are shown in figures 9(a) and 9(b), respectively (see coloured pages), and the main impression is one of massive variability of house type in spatial terms. There is almost a complete mix of types everywhere for both types of equation, thus implying that the relative evenness and similarity of the probability profiles gives much greater weight to the lower probabilities in each choice situation than would be the case in a real context. Little spatial pattern can thus be discerned and this suggests that random simulations based on discrete choice models are likely to produce too little spatial discrimination if predicted in this way.

The deterministic simulations which involve equation (30) are shown in figures 10(a) and 10(b) (see coloured pages) for the full, and inner-outer models, respectively. Very clear spatial patterns emerge this time which show the characteristic structure of residential land use in London, but there is little difference between the two sets of models. The clearer of the two patterns is figure 10(a) based on the full model but there is a ring of purpose-built flats between the terraced and detached or semidetached areas which is unexpected.

In figure 10(b), purpose-built flats are closer in towards the CBD. Note that in the simulations the total number of house choices is not scaled in any way to reflect the scale of housing in London; thus this represents an additional prediction from the model. The patterns in general, though, are very plausible, reflecting flats, terraced, and detached or semidetached houses at increasing distance from the CBD, with the distribution of purpose-built and converted flats clearly characterising the market for flats in London. One limitation of the deterministic model is that it does not pick up the degree of local variation one might expect, but a more detailed database might resolve this.

Last, we have begun to experiment with these simulations. Running the models at  $r > 4$  requires the use of a 6502 second processor because the amount of program memory required explodes as a result of the recursion, and we have run the model for  $r = 6$ , which took 24 hours. Level of recursion does affect the pattern we get, but generally these help us to improve the recursive geometric generation, not the models themselves. Simple policy-predictive runs of the simulations are possible, for the input data are easy to update. One could assume a process of aging and renewal, varying according to simple rules and policies, which would then enable a pseudodynamic simulation to be developed. A series of images of the typical house types in London over the next fifty years could be generated in this way. But these are for the future and, in any case, there are many other improvements to be made before then. These will form our conclusions.

### Conclusions

The ability to display the overall pattern produced by models with an implicit spatial dimension is a clear advantage of the large-scale simulations adopted here. But these need not be generated within a fractal framework. Simulation could proceed by examining each pixel in turn and building up urban structure in this way on a regular spatial grid. Nevertheless, fractals do generate realistic images and one of the goals of this work is to make abstract models more visually intelligible and acceptable, and in this way, the fractal framework seems promising. As such, the technique is one of generating spatial realism and it clearly depends upon the display devices used.

The main problem emerging from this paper, however, relates to the development of a more consistent modelling strategy which can be effectively incorporated into the hierarchical method used to structure the simulation. We have already indicated what is involved: in essence, the hierarchy guiding the fractal simulation should be based on characteristics of the city, and this clearly relates to the type of explanation and modelling required. Discrete choice models show promise here, but so do sequential and nested approaches involving entropy models.

In fact, a more fundamental strategy may be actually to explore the possibility of underlying land-use models which are themselves fractal. For example, the sorts of terrain model explored by Goodchild (1982) and the image-processing techniques developed by Pentland (1984) are suggestive of the types of stochastic model that might underly the structure of land use. There are difficulties in that some of the patterns are discontinuous, but it is worth exploring how such ideas could be used to link what we already know about land use, central place, and rank-size together in a fractal framework. With respect to discrete choice models, there may even be the possibility of fractal interpretation of the underlying mechanisms which give rise to various forms of logit and probit models and there is clearly a possibility that questions of nesting and aggregation might be reconciled with ideas about recursion and hierarchy. In fact, in this paper, the whole question of the spatial basis of

discrete choice models has emerged as problematic and this suggests that further research on spatial aggregation and discrete choice is worthy as an end in itself, notwithstanding any fractal interpretations which might emerge.

We need to improve the more practical aspects of the project, mainly the graphics computers that are available to us and the database we have access to. We are at work now using a graphics device with a much higher resolution and this should enable more maps to be held in screen memory, and hence allow more data to be used. We hope to reprogram the simulation in Pascal, instead of structured BASIC, thus enabling the geometric data structure to be better handled. We intend to continue working with the EHCS data, but hope to mount a project on the measurement of fractal pattern in land use using cartographic databases. But much will depend on the availability of data which will continue to constitute a limitation to this work.

Many other speculations are possible about where we are headed. An interesting project would be to examine the extent to which regular, nonrandom fractal patterns built from cell-space models (Tobler, 1979; Couclelis, 1985) could be used as first approximations to city patterns. We also need to consider how such simulations might be made dynamic, especially as there is an obvious dynamic process underlying a model in which age acts as an independent variable. In one sense, our models might already be seen as explaining urban structure in terms of time and space, age and distance, and our earlier comments on possible policy simulations endorse this. In particular, the question of redevelopment is central to residential location, and any dynamic extension to the framework should enable such processes to be captured. These ideas suggest a broad research programme, but at present we require a stronger link between exploration and simulation through ideas concerning spatial hierarchies, and this should enable a clear research strategy to be mapped out to improve upon and extend the realism of urban models.

**Acknowledgments.** This paper was first presented at the 17th Annual Meeting of the British Section of the Regional Science Association, University of Manchester, 4-6 September 1985. Beryl Collins typed the manuscript.

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