BUILDING RESEARCH & INFORMATION (2009) **37**(5-6), 455–467

Routledge
Taylor & Francis Group

RESEARCH PAPER

Wall area, volume and plan depth in the building stock

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The relationship between the volume of a building and its wall area follows an allometric rule that implies that building shape distorts to capture as much surface area, hence natural light, as possible as it increases in size. For a sample of house plans, Bon in 1973 established that the relationship between wall area W and volume V scaled as $W \sim V^{0.77}$, and Steadman in 2006 demonstrated a similar relationship for his archetypal building. Empirical work in Cambridge and Swindon, UK, also revealed a similar allometry as measured by the depth ratio based on V/W, which provides a direct measure of the way building shapes become distorted with increasing size. This paper demonstrates positive allometry for building blocks taken from a large urban database (approximately 3.2 million blocks) for Greater London which is constructed from Ordnance Survey building footprint data augmented by remote sensing light detection and ranging (LIDAR) height data. For the domestic and then non-domestic stock, the blocks are categorized into eight bands and the depth ratios in six inner-London boroughs including the City, which is the financial quarter, are then examined. This is demonstrated in two ways – first, from the depth ratio; and second, from fitting allometric relationships to the band data. The allometric coefficients converge to values of around 0.77, thus confirming the magnitude of Bon's relationship, implying that positive allometry not only is a feature of small samples of houses and archetypal buildings, but also is more generally the case for real building databases at the very largest urban scales.

Keywords: allometry, building envelope, building geometry, building stock, built form, depth ratio, plan depth, surface area, volume

La relation entre le volume d'un bâtiment et sa surface de paroi suit une règle allométrique qui implique une distorsion de la forme du bâtiment afin de capturer le plus possible de superficie, et donc de lumière naturelle, au fur et à mesure qu'il augmente en taille. Bon a établi en 1973, pour un échantillon de maisons sur plan, que la relation entre une surface de paroi W et un volume V donnait une échelle égale à $W \sim V^{0.77}$, et Steadman en 2006 a démontré une relation similaire pour son bâtiment archétypal. Les travaux empiriques réalisés à Cambridge et à Swindon, au Royaume-Uni, ont également révélé une allométrie similaire mesurée en utilisant le rapport de la hauteur à la longueur basé sur V/W, qui fournit une mesure directe de la manière dont les formes des bâtiments subissent une distorsion lorsque leur taille augmente. Cet article démontre l'allométrie positive d'immeubles pris dans une grande base de données urbaine (immeubles d'environ 3,2 million) du Grand Londres qui a été élaborée à partie des données de l'Ordnance Survey [Service Cartographique National], relatives aux surfaces au sol des immeubles, augmentées des données relatives à leur hauteur obtenues en utilisant le LIDAR (système de télédétection et de télémétrie par ondes lumineuses). Pour le parc d'immeubles résidentiels, puis non résidentiels, les immeubles ont été catégorisés en huit tranches et les rapports de la hauteur à la longueur dans six arrondissements du centre de Londres, y compris la City, qui est le quartier financier, ont ensuite été étudiés. Cette allométrie positive a été démontrée de deux manières – tout d'abord par le rapport de la hauteur à la longueur; et deuxièmement, en adaptant les relations allométriques aux données de chaque tranche – de sorte que les coefficients allométriques convergent vers des valeurs d'environ 0,77, confirmant ainsi l'ampleur de la relation de Bon, ce qui implique que l'allométrie positive est non seulement un élément caractéristique de petits échantillons de maisons et d'immeubles archétypaux, mais que, plus généralement, elle se vérifie également dans le cas de bases de données d'immeubles réels aux échelles urbaines les plus grandes.

Mots clés: allométrie, enveloppe du bâtiment, géométrie du bâtiment, parc immobilier, forme construite, rapport de la hauteur à la longueur, profondeur en plan, superficie, volume

Building Research & Information ISSN 0961-3218 print/ISSN 1466-4321 online
http://www.informaworld.com/journals
DOI: 10.1080/09613210903152531

Depth and allometry in building geometry

The advent of large-scale three-dimensional (3D) virtual models of cities has opened up new opportunities for research on urban built form. In this paper we describe some geometrical analyses from our Virtual London model, developed primarily so that future changes and plans might be visualized for a range of activities involving general dissemination and public participation (Batty and Hudson-Smith, 2005). The model uses the Ordnance Survey's digital MasterMap for its topographic base, and remotely sensed light detection and ranging (LIDAR) data on a 1 m grid spacing for the heights of buildings or parts of buildings. The model currently extends out to London's M25 orbital motorway, and comprises some 3.2 million 3D blocks. Figure 1 shows part of the model covering an area in the financial quarter of the city including the River Thames.

One geometrical property of building stocks that has been studied in the past on relatively small samples is the ratio of volume-to-surface area – or else the ratio of volume-to-external wall area, ignoring roofs. The first person to make such an analysis was Ranko Bon, who was a member of the Philomorphs, an interdisciplinary seminar at Harvard in the 1960s, which also included the palaeontologist Stephen Jay Gould and the geographer Michael Woldenberg. The Philomorphs were interested, among other morphological topics, in extensions of the biological concept of *allometry* to social systems, cities and buildings, as presented in a special issue of *Ekistics* in 1973 (Dutton, 1973a). Allometry describes the ways in which organisms change shape as they increase in size during development in order to preserve certain geometrical properties important for physiological function. The ratio of surface area-to-volume is one of these properties, since it affects heat loss or gain through the skin. Allometric effects can also be seen in comparisons of the adult forms of animals of different sizes between different species and genera.

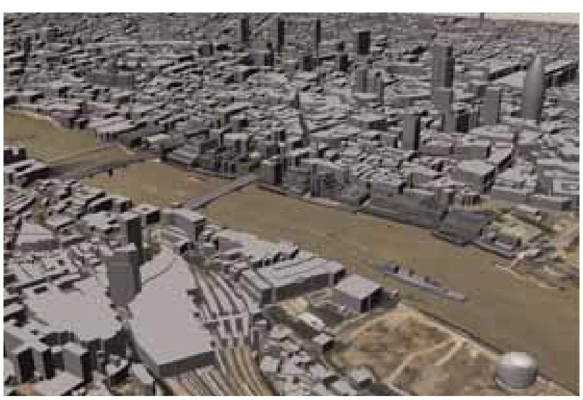

Figure 1 Virtual London: detail along the River Thames

In general, one should be cautious about drawing analogies – which can be treacherous – between animal physiology and the functioning of buildings (Steadman, 2008). Individual buildings do not 'grow' (although they can be extended). The forms of animals are in many ways more flexible than those of buildings. One can speak figuratively of the 'metabolism' of buildings, but one must be clear exactly what is meant in terms of physical processes. The forms of many buildings are limited by a general requirement for natural light and natural ventilation; but in other cases these constraints are broken with the use of artificial lighting and air-conditioning. There can be allometric relationships – as Bon showed – between the lengths of circulation routes in buildings and the floor areas they serve; but again the circulation of people is hardly the same as the circulation of the blood, with which architects have sometimes drawn analogies. All this said, however, there can be no doubt that allometric relationships exist between the volumes and surface areas of buildings, at least under certain conditions, as Bon showed and this paper will confirm.

Bon took a sample of 40 residential buildings of greatly varying dimensions, from Neolithic and Egyptian huts to grand hotels and high-rise apartment blocks, taking in mobile homes, modern houses and mansions along the way (Bon 1972a, 1972b, 1973). These were selected at random from R. Martin Helick's atlas *Varieties of Human Habitation* (Helick, 1970). It is important to emphasize that these were all *detached* buildings. The same is not true for our London measurements, a fact that has significant consequences for the results, as will be explained below. Bon measured volume V and exposed wall area W in each case. The latter he defined as the 'exterior wall surface area of the actual living compartments generalized to a plane, not including non-habitable spaces, such as attics' (Bon, 1972b, p. 10). The basic allometric relation between wall area and volume is:

$$V \sim W^{\alpha}$$

where α is the allometric coefficient. Figure 2 shows Bon's plot of log V against log W. The strong linear correlation and the value of α confirms that there is indeed a marked allometric effect: the ratio of wall surface-to-volume increases faster than the simple increase in surface area associated with an increase in the volume of a rectangular building of unchanging shape. The forms of larger buildings are not in general simple magnifications of the forms of small buildings. Instead, the bigger structures become flattened and elongated, in either the horizontal direction into slabs or in the vertical direction into towers. Recently, Steadman (2006) has replicated Bon's empirical results theoretically, by means of an

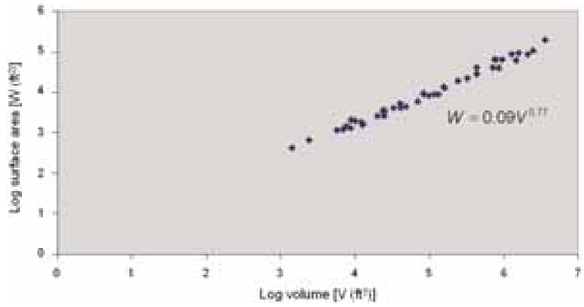

Figure 2 Measurements of wall area *W* and volume *V* for a sample of 40 residential buildings, replotted on a log–log scale from data given by Bon (1972a)

'archetypal building' from which many built forms of varying sizes can be generated.

Why precisely should this allometric effect occur? It is found because Bon's examples are all residential buildings in which the great majority of 'habitable' rooms – living rooms, kitchens, bedrooms – are daylit via windows. (There may also be some artificially lit corridors, small storerooms, or bathrooms in the interior.) The windows also serve of course to provide natural ventilation and views of the exterior. This means that the plans of these buildings *must in no place be more than two habitable rooms deep*, because if they were three or more rooms deep, the rooms in the centre could not have windows. The plans might be one room deep, and this is indeed found in larger detached houses. But such plans are less common in medium-sized and small houses and flats, for reasons we will come to shortly.[1]

Now the fact is that most habitable rooms in modern dwellings have dimensions in plan of around 3 or 4 m. (Obviously there are many exceptions.) This we may assume has something to do with the typical space requirements of domestic activities and their associated furniture and equipment. In dwellings that are two rooms deep this would imply a total plan depth of around 7 or 8 m. Brown and Steadman (1991) made a survey of a random sample of 300, mostly 19th- and 20th-century houses and flats, in Cambridge, UK. They measured their depths in plan in every case (ignoring minor back extensions) and obtained a mean value for the whole sample of 7.4 m. Breaking down the sample by house types, they found mean depths of 7.2 m for terrace houses, 7.7 m for semi-detached houses, and 7.4 m for flats, as illustrated in Figure 3. These they showed were the consequences of placing pairs of habitable rooms with dimensions of 3–4 m, back to back. Detached houses were somewhat shallower, with a mean of 7.0 m, because some were, at least in part, just one room deep.

As more daylit rooms are added on the same floor level, so a building must become elongated in order to preserve the two-room depth. The architect Roger North recognized this fact as long ago as the 17th century in his writings on the design of country houses (Colvin and Newman, 1981, p. 9). He says that for a small house, a square plan will serve, but in a 'great pyle' the plan 'must be spread for air and light'. We can understand the relationship between volume, wall area and plan depth by considering a simple rectangular block as shown in Figure 4. The depth is *d*, the length *l*, the number of storeys *n* and

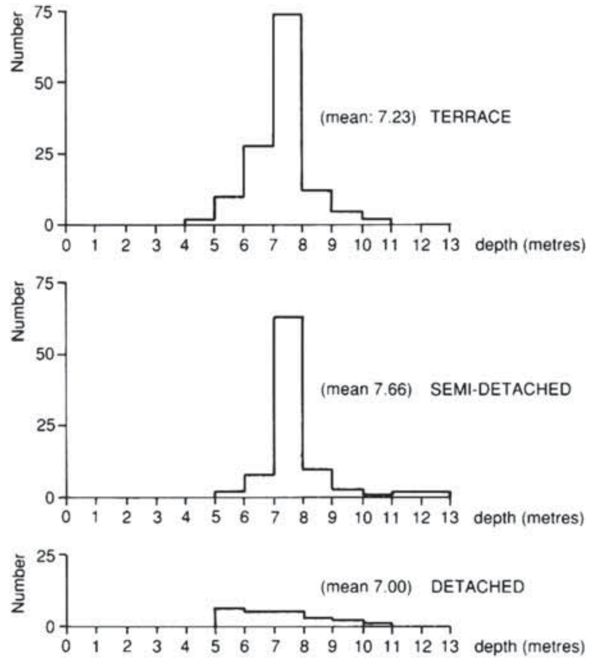

Figure 3 Statistics from a representative survey of 300 dwellings in Cambridge, UK, made by Brown and Steadman (1991) showing distributions of overall plan depth (m) for terrace, semi-detached and detached houses

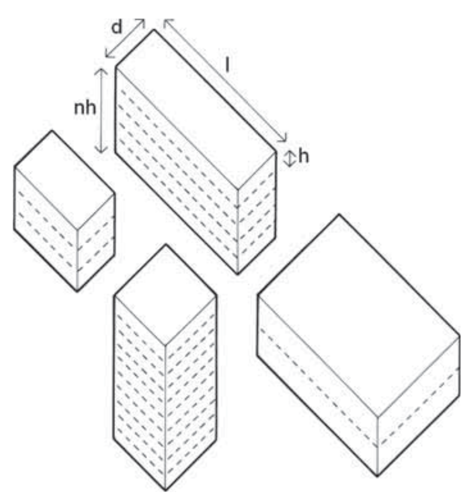

Figure 4 Building block with depth *d*, length *l*, number of storeys *n* and storey height *h*

the storey height h. Supposing we ignore the short end walls, in which case total wall area:

$$W = 2nlh$$

$$\text{Volume} = dnlh$$

Thus:

$$V/W = (dnlh)/(2nlh) = d/2$$

On this basis, the ratio of volume-to-wall area for a long block is dependent simply on plan depth, and is not affected by changes in the length or height of the block.

On the other hand, for small buildings the areas of the end walls can be significant. Consider a free-standing block where $l = 5$ m, $h = 5$ m and $d = 7$ m. This could be a small detached house. When the areas of the side walls are included as well as the back and front walls, then $V/W = 1.46$. Let us now add more houses of the same size to form a terrace. The value of V/W *rises* progressively, as shown in Table 1, until when the terrace is long, V/W approaches 3.5. Note that the depth of 7 m is maintained throughout.

A block of this kind could be extended indefinitely in length in a straight line, or it could be cut up and rejoined into patterns of branching wings or courts. It could also be enlarged of course by adding more floors. This is how the ratio of wall area-to-volume can remain more or less the same while residential buildings change their forms, as they increase in size.

The relationship between V, W and $d/2$ is evident in Bon's data when his actual values are plotted. Two graphs at different scales show the smaller buildings in Figure 5 and the larger buildings in Figure 6, respectively. A line corresponding to a value of 3.5 for the ratio V/W, implying a plan depth of around 7 m, is superimposed on both graphs. See how buildings in the middle of Bon's size range lie near this line. The huts and small houses have values of V/W below 3.5. This does not necessarily mean that their plan depths are less than 7 m, although this *could* be the case especially in the smallest examples. It reflects the fact that these are small detached buildings where the areas of the side walls are significant, as demonstrated. Notice by contrast in Figure 6 how several of the apartment blocks and hotels lie below the $V/W = 3.5$ line. This *must* mean that they are deeper than 7 m in

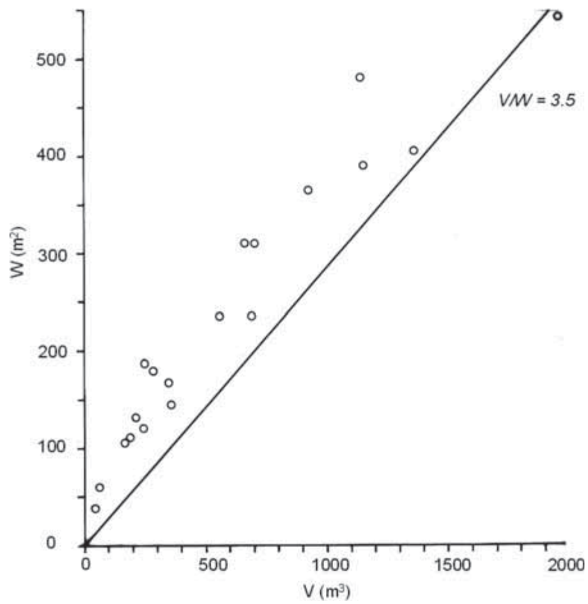

Figure 5 Measurements of wall area W and volume V for the sample of small residential buildings given by Bon (1972a) (compare with Figure 2). This graph shows actual values for the smaller buildings. The heavy line marks a value of 3.5 for the ratio V/W, implying a plan depth of around 7 m. Some of these dwellings may be shallower than 7 m

plan, going up to around 14 m at the extreme. Such big buildings are likely to have central corridors, and possibly also internal windowless bathrooms and kitchens.

The characteristic depth of medium-sized houses and flats is *not*, however, determined by the maximum distance to which daylight can penetrate these domestic buildings from the two sides (depending on the sizes of windows, the level of lighting required at the centre of the plan, and some other factors). The depth is constrained by the requirement for daylight *together with* typical domestic room sizes. The absolute limit of depth for daylighting is greater, as we shall see shortly.

This argument explains then why many domestic buildings are not much deeper in plan than 7 or 8 m. We also find, however, that most dwellings tend not to be much shallower than this, and are not in general just one room deep. Why should this happen? We can see the reasons by considering some worked examples of simple rectangular blocks. Figure 7 shows a five-storey block with $d = 7$ m, $l = 25$ m

Table 1 V/W for terraced rows with increasing numbers of houses

Number of houses	1	2	3	4	5	6	7	8	9	10	100
V/W	1.46	2.06	2.39	2.59	2.73	2.84	2.92	2.99	3.03	3.07	3.45

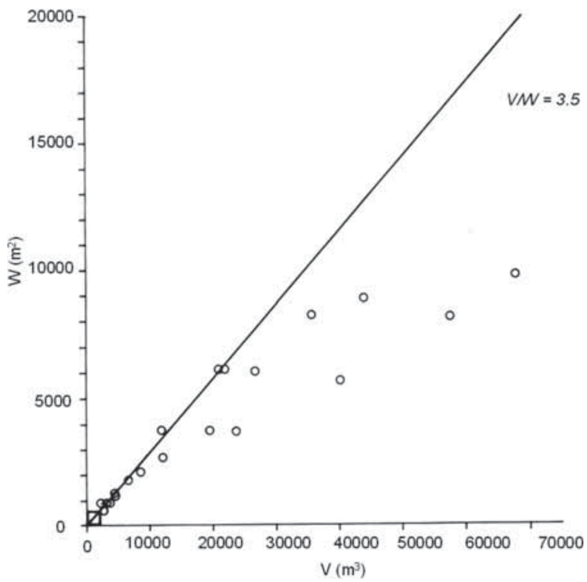

Figure 6 Measurements of wall area *W* and volume *V* for the sample of large residential buildings given by Bon (1972a) (compare with Figure 2). This graph shows actual values for the larger buildings. The heavy line marks a value of 3.5 for the ratio, implying a plan depth of around 7 m. Many of these dwellings are deeper than 7 m

and $h = 3$ m. (The height and length are arbitrarily chosen and do not affect the comparisons that follow.) The combined area W of the long walls is 750 m^2 and the volume V of the block is 2625 m^3 giving a value for V/W of 3.5 (half the plan depth).

Figure 8 shows a thin block just 3.5 m deep, each floor consisting of a single row of domestic rooms. Otherwise all dimensions are as in Figure 7. Now V/W takes the value of 1.75. The ratio of volume-to-wall area can have important practical consequences for the rate of heat loss from buildings per unit volume via the walls. It can also affect the buildings' capital costs. In order for the 3.5 m-deep block to provide as much volume as the 7 m-deep block of Figure 7, it must be made twice as long (50 m), involving a doubling of W to 1500 m^2, as shown in Figure 9. The same volume is contained within roughly twice the

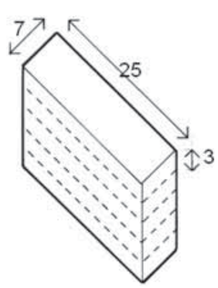

Figure 7 Five-storey building block with depth 7 m, length 25 m and storey height 3 m. The value of V/W (discounting the end walls) is 3.5

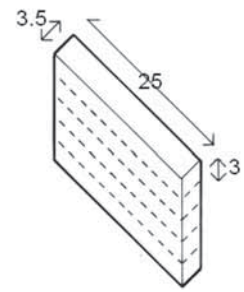

Figure 8 Five-storey building block with depth 3.5 m, length 25 m and storey height 3 m. The value of V/W (discounting the end walls) is 1.75

area of expensive exterior wall through which heat may be lost. North recognized this problem when he spoke of the costs of 'too much spreading' in mansions with plans that are just one room deep. Not only is there a great 'charge of walls', in North's phrase, but also a need for 'long entrys and passageways' (Colvin and Newman, 1981, p. 69). Imagine circulation routes running along the deep and shallow blocks of Figures 7 and 9. The route in the shallow block must be approximately twice as long as that in the deep block, serving the same floor area.[2]

Thus, different 'generic functions' of architecture, acting in opposite directions on the built forms of typical medium-sized dwellings, tend to produce the regularities in plan depths observed by Brown and Steadman (1991). Daylighting and room sizes tend to keep the buildings from being made much deeper than 7 or 8 m, and capital and running costs, and the costs of circulation space, tend to keep them from being made much shallower.

Moving now to non-domestic buildings, we find that similar considerations apply, although typical values for plan depth are greater. Figure 10 plots values for the depth in plan of 19 office buildings in Swindon,

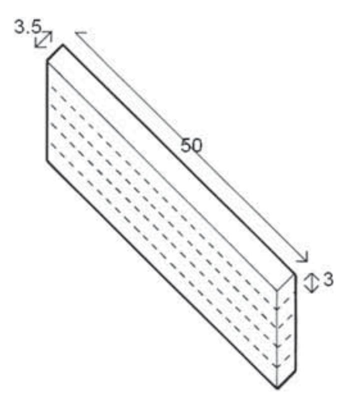

Figure 9 Five-storey building block with depth 3.5 m, length 50 m and storey height 3 m. This has the same volume as the block shown in Figure 7

Wiltshire (Steadman *et al.*, 1993, p. 84). The graph shows total areas of floor space in these buildings in blocks or wings of different depths. Notice how the distribution has two peaks, at 14 m and 22 m, respectively. The lower value corresponds to daylit buildings, and the higher value to buildings that are air-conditioned and which rely in their central zones on permanent artificial lighting.[3]

It has been observed since the 19th century that 7 m is about the furthest distance from window walls that daylight sufficient for office work will penetrate (the exact distance depending again on many factors including window sizes, ceiling heights, the colours of interior surfaces, and whether or not there are obstructions outside the windows). In the American literature on office planning of the 1880s and 1890s, for example, it was frequently said that space more than 20–25 feet (6–7.5 m) from windows would be difficult or impossible to let (Willis, 1995, pp. 24–30). This rule of thumb continues to be applied today. For example, the LT Method for calculating energy use in commercial buildings divides their plans into two zones. There is a 'passive' perimeter zone, whose depth is twice the floor-to-ceiling height (i.e. typically 6 m) assumed to be daylit. And – where this exists – there is a 'non-passive' core zone, further than this distance from the windows and assumed to be artificially lit and ventilated, which would in many cases also need to be cooled (Baker and Steemers, n.d.).

Going back to the Swindon office buildings of Figure 10, the peak in plan depth of 14 m could correspond then to two rows of daylit cellular offices of around 6–7 m depth, flanking a central 2 m corridor – or else to a fully daylit open plan. (Notice in Figure 6 how Bon's largest residential buildings, the apartment blocks and hotels, also have plans up to 14 m deep at a maximum.) In Figure 10 it is possible to see a very small proportion of floor area with 7–8 m depth, corresponding to office buildings or wings of buildings which – perhaps because of the constraints of their sites – receive daylight from one side only, and comprise just one row of rooms plus a corridor.

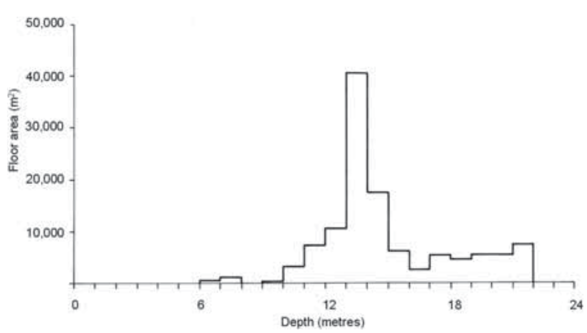

Figure 10 Plan depths of 19 office buildings in Swindon, UK, from a survey reported in Steadman *et al.* (1993). The graph shows the total floor area (m^2) in the sample with the given plan depth (m)

Building depths from the Virtual London block model

The Virtual London model makes it possible to calculate volumes V and external wall areas W of buildings – both domestic and non-domestic – on an unprecedented scale. The volumes of building blocks can be calculated relatively simply by multiplying the areas of their footprint polygons by mean heights derived from the LIDAR data. Measurements of exposed external wall areas are more complex since it is necessary to distinguish these from unexposed party walls shared by adjacent building blocks.

The relatively straightforward procedures for calculating topological relationships within a geographic information system (GIS) can help solve this issue. Different types of topological relation can be expressed as lists of features (e.g., an area is defined by the arcs comprising its border). In this way, the walls of the building footprints can be categorized as 'children' of their 'parent' polygons. The walls can be assigned heights, and by spatially analysing the polygons that they adjoin it is possible to determine if they are walls that face onto a courtyard, or onto the street, or are party walls (walls which divide terraced or semi-detached properties). Sometimes these party walls divide properties of different heights, in which case there is an area of the party wall that rises above the roofline of the lower property and becomes 'exposed'. The exposed areas of these walls are also taken into account when making the calculations.

The Ordnance Survey MasterMap data can be combined with the Generalized Land Use Database (GLUD), which contains classifications of land cover. For building footprints these draw a basic distinction between domestic and non-domestic buildings. Measurements of V and W and values for V/W are given here for six selected boroughs within London. Three of these – the City of London, Westminster, and Tower Hamlets – cover between them much of the capital's financial and major office districts, and are the most densely developed. The remaining three – Hackney, Islington, and Camden – are predominantly residential with some retail, commercial and industrial uses. Table 2 gives total numbers of blocks, domestic and non-domestic, broken down by borough. For each borough, results have been grouped into ten approximately logarithmic sized bands by volume. Table 3 gives the V/W results for domestic and Table 4 for non-domestic. Notice that these statistics relate to *building blocks*, each of which corresponds to a single ground polygon in the map. These blocks might or might not correspond to

Table 2 Numbers of building blocks, domestic and non-domestic in the six boroughs

Borough	Domestic	Non-domestic	All
City	2 414	2 577	4 991
Camden	38 683	12 956	51 639
Hackney	43 756	12 233	55 989
Islington	39 850	11 504	51 354
Tower Hamlets	34 650	15 287	49 937
Westminster	37 859	16 165	54 024

'buildings' understood in some architectural or constructional sense. Many are just small parts of buildings. This is critical to what follows.

The value of V/W can give an indication of the mean plan depth of the building blocks in question, as explained. It might be asked why could plan depths not be measured directly on the Virtual London model? The depth of a theoretical rectangular block is given by definition, but the problem with real buildings is that they can take many shapes in plan where depth is not so easily defined. Imagine a simple L-shaped block. The depths of the wings themselves are known, but plan depth is undefined in the rectangular zone where the two wings meet. Where buildings have non-orthogonal, curving or indented outlines in plan, the problem becomes yet more difficult.

One feasible approach is to draw contour lines within the plan, offset at some constant distance from the window walls. It is then possible to measure the floor area within x m of the perimeter. This is the approach taken in effect by the LT Method for defining passive perimeter zones. Where the plans of buildings are represented in a GIS, the method can readily be automated. Figure 11 shows examples of analyses of building blocks in Swindon made by the present authors with offset contours at 1 m spacing. The distribution of floor area between successive contours gives a profile that can be seen as in some sense characterizing plan depth.

Here, however, we stick with the simple ratio V/W. In Table 3 showing domestic buildings in London, the overall figures give totals for V and W and the value of V/W for the whole of each borough. In effect, we are lumping all blocks together and considering

Table 3 Depth ratios V/W for the domestic stock

Size band (m^3)	Westminster	City	Hackney	Islington	Tower Hamlets	Camden
0–3	0.037	0.080	0.104	0.113	0.022	0.076
3–10	0.300	0.360	0.439	0.411	0.485	0.344
10–30	0.603	0.467	0.829	0.726	0.785	0.738
30–100	1.030	0.897	1.152	1.191	1.158	1.110
100–300	2.112	1.438	2.427	2.326	2.438	1.887
300–1000	3.317	2.954	3.412	3.396	3.530	3.201
1000–3000	4.358	4.213	3.689	3.775	3.709	3.628
3000–10 000	5.559	5.101	4.585	5.220	4.935	4.784
10 000–30 000	7.104	7.179	5.917	7.857	5.967	6.750
$\geq$ 30 000	11.229	11.247	6.893	16.152	11.139	13.762
Overall	4.355	5.847	3.453	3.503	3.755	3.534

Table 4 Depth ratios V/W for the non-domestic stock

Size band (m^3)	Westminster	City	Hackney	Islington	Tower Hamlets	Camden
0–3	0.239	0.056	0.105	0.180	0.117	0.131
3–10	0.410	0.256	0.503	0.449	0.428	0.390
10–30	0.777	0.479	1.107	0.856	0.882	0.814
30–100	1.207	0.906	1.298	1.283	1.115	1.233
100–300	1.671	1.602	1.895	1.839	1.909	1.814
300–1000	3.593	3.088	3.047	3.282	2.933	3.339
1000–3000	4.871	4.918	4.195	4.210	3.780	4.279
3000–10 000	6.056	6.004	5.010	5.429	5.223	5.390
10 000–30 000	7.402	7.774	6.447	7.011	6.674	7.091
$\geq$ 30 000	12.086	14.180	12.911	12.802	17.148	13.598
Overall	7.284	10.061	4.964	5.709	7.012	6.524

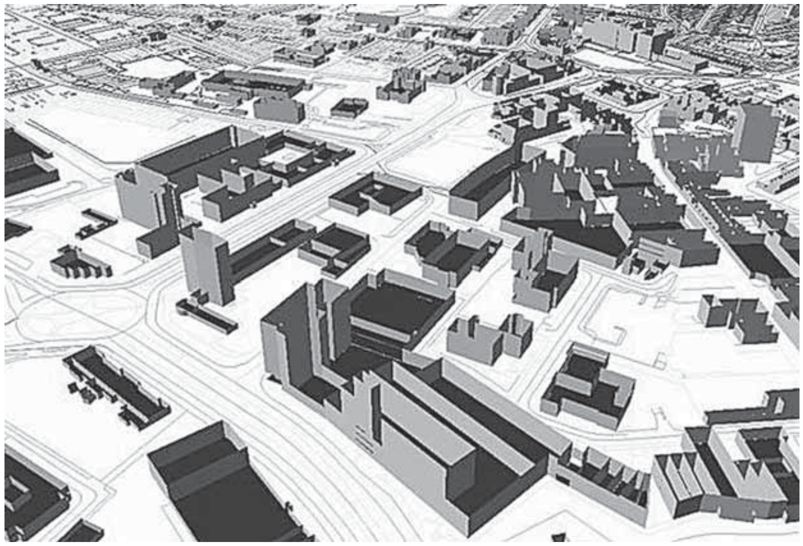

Figure 11 Buildings in Swindon, UK, modelled in ArcView GIS, with internal plan contours offset from the window walls at 1 m spacing. The 1 m bands are coloured in darker shades with increasing distance from the windows

them as if they were amalgamated into one giant 'building' whose depth is specified by V/W. In the residential boroughs of Hackney, Islington and Camden, V/W is close to 3.5, implying a mean plan depth of 7 m, as would be expected. In the more densely developed boroughs the value rises to 3.76 in Tower Hamlets, 4.36 in Westminster, and 5.85 in the City. This must be due to the presence here of larger numbers of flats – typically with greater depths like Bon's examples in Figure 6 – and fewer houses. There will also be buildings in these three boroughs in which offices and flats are combined.

The breakdown by size bands is more difficult to interpret. We should re-emphasize that the statistics here relate to building blocks, not buildings: specifically, these are prisms whose bases are the polygons into which building footprints are digitized. One building may be digitized with numerous polygons corresponding to sections or wings of different heights, stair and elevator towers, porches and lean-tos, and so on. This explains the fact that many building blocks are found in the very smallest size bands, between zero and 30 m^3. In these domestic statistics, these are not dwellings but parts of dwellings. There is no reason why these blocks should extend from the fronts to the backs of buildings, or why therefore they should obey the allometric relationships discussed by Bon (1973). An average terrace or semi-detached house might have a volume of around 250 m^3. We find that the values of V/W for the middle bands in Table 3 lie between 2.0 and 4.0, suggesting that these do indeed relate to entire houses digitized as single ground polygons. The higher size bands must relate for the most part to blocks of flats, or sections of such blocks. Here V/W goes up to 7 or 8, consistent with Bon's data for larger American apartment buildings and hotels. It is difficult to explain why there are values of V/W between 11 and 16 in the 30 000 m^3 plus band. These are improbably deep buildings for domestic use. They could be caused by misclassifications in the GLUD.

Table 4 gives equivalent results for non-domestic buildings. We could expect more variation in V/W here, since these will include not just office and institutional buildings such as hospitals and schools, many of which can be expected to be daylit, but also deeper air-conditioned offices. There would also be factory sheds, warehouses and supermarkets, which could be either single storey and top lit, or lit entirely by artificial light. The plan depths of these latter structures could be extremely large. Looking at the overall V/W values for entire boroughs, however, we find values of 7.01 and 7.28 in Tower Hamlets and Westminster, respectively, suggesting a predominance of 14 m deep office and other daylit buildings. In the City of London the overall value for V/W goes up to 10.06, indicating the presence of a proportion of deep-plan artificially lit offices. In the three largely residential boroughs, by contrast, the mean values are 4.96 (Hackney), 5.71 (Islington) and 6.52 (Camden). Here much of the non-domestic stock is made up from smaller offices and shops, many of them comparable with domestic buildings in their built forms.

Within the volumetric size bands, the bulk of the accommodation in all boroughs falls in the ranges from 1000 m^3 upwards, implying building blocks with more than 300 m^2 of floor area. Between the 10 000–30 000 m^3 and the 30 000 m^3 plus bands, V/W jumps typically from around 7 up to a value of 12 or greater. It is in this topmost size band that both the very largest office buildings and the biggest industrial and retail building blocks are likely to be concentrated. Notice the value of 17 in this band in Tower Hamlets associated with the major offices concentrated in Canary Wharf. The land cover categories in Ordnance Survey Address Layer 2 data (which can be used in conjunction with MasterMap) distinguish a number of types of building within the broad classification of 'non-domestic'. These include Education, Industrial, Leisure, Office, Office Mixed Use, Restaurant/Public house, and Retail. In future work the statistics in Tables 3 and 4 might be broken down into these categories in order to understand better the values of V/W by size band. A preliminary analysis of such a disaggregation is presented in Batty *et al.* (2008).

A statistical analysis of allometry in the London data

So far we have examined the relationship between wall area W and volume V in terms of the ratio V/W and found that as the building blocks become bigger in wall area and volume, the depth in plan increases significantly from an average about 3.5 m for domestic buildings in Islington to just over 10 m for non-domestic buildings in the City. This is an immediate outcome of their allometry and in this section we will generate aggregate statistics consistent with these depth ratios in the spirit pioneered by Bon (1973) which goes back to the original insights of Huxley (1924).

The Euclidean relationship $V \sim W^{\alpha}$ between area and volume is given as $W \sim V^{2/3}$, where it is assumed that there is no deformation of the surface area of the object as its volume increases. If area and volume are calculated from some length measure L, then the standard Euclidean equations hold as $W = L^2$ and $V = L^3$ from which this relationship can be derived directly. It is also clear that the ratio $V/W = L$ and from this, as the object or building gets larger, the surface (or wall) area declines in proportion to this unit linear measure. In fact as Bon (1973) first demonstrated for his sample of houses, $W \sim V^{0.77}$, that is the wall area does not decrease as fast as the linear measure which implies that the shape of the building deforms to capture more natural light from its surface. In this case as is implied in Figures 5 and 6, the ratio increases as $V^{0.23}$. To demonstrate this effect, if we take a cubed block with $V = 6 \times 6 \times 6 = 216$, then $V/W = V^{0.23} = 3.443$ in contrast to the Euclidean case where there is no deformation of wall area which gives a ratio of 3. If we double the size of the unit length to a block with $V = 12 \times 12 \times 12 = 1728$, this gives a Bon ratio of 5.554, compared with the Euclidean ratio of 6.000 where there is no deformation of the wall area. These values are consistent in terms of magnitude with the results presented in the last section, and with the depth calculations for the terraced block in Table 1.

In fact, previous work with respect to deriving allometric relationships between surface area and volume for entire cities is sketchy and nowhere comprehensive. Nordbeck (1971) was amongst the first to examine the relationship between ground area A and population P of cities in Sweden. He found that populations were not distorted, filling their space according to the standard Euclidean relationship; that is for the largest 1800 towns in 1960, he found that $A \sim P^{0.664}$ where about 90% of the variance was explained. This coefficient only changed marginally to 0.650 when he did the same analysis for data for the same towns in 1965. Batty and Longley (1994) in their work on relating allometry and fractals, however, found that there was considerably more distortion (of the area) when they compared the 70 largest towns in the county of Norfolk with their areas using 1981 population data. Here they found that $A \sim P^{0.959}$, implying that population does not scale as a volumetric measure but simply directly with area. In small towns, of course, where there are few high buildings and relatively uniform densities, this finding makes sense and is consistent with other work by Woldenberg (1973) and Dutton (1973b).

Until we extracted the wall areas from the London database, we could only previously relate the footprint area of building blocks to their volumes and although we computed allometric relationships between these areas and volumes, these results did not relate to detached buildings and simply worked with the blocks as defined in the original Ordnance Survey MasterMap building footprints from which the databases are constructed. Nevertheless, we have computed equivalent allometric relationships between volume and footprint for all buildings in the residential and commercial classes as defined from land uses in the MasterMap data. The relationships derived for residential (domestic) are $A \sim V^{0.755}$, for commercial (non-domestic) $A \sim V^{0.834}$, and for all buildings $A \sim V^{0.772}$. These show more distortion than might be expected but this is not for wall area, and the points contain a very large number of building blocks that might be considered to be parts of buildings (Batty *et al.* 2008).

What we have done here is to fit the general allometric relationship $W \sim V^{\alpha}$ for the ten size classes that we defined to examine the volume-to-wall ratios in the last section. We have regressed $\log W$ on $\log V$ to determine the allometric coefficients for the domestic and non-domestic size bands for each of the six boroughs. Then we have summed the data for the domestic and non-domestic stock, finally summing domestic and non-domestic for all boroughs. We have normalized the size bands in each case by taking an average of the building block size in terms of wall area and volume; that is, we have divided each band which consists of all wall area and volume of all the blocks in the band by the number of blocks. In essence, the allometric relationships that we derive are thus based on an 'average building block' for each band in terms of wall area and volume.

The results we derive are shown in Table 5 where we have excluded the first category – buildings between zero and 3 m^3 in volume – from the analysis, thus reducing our number of bands to nine. We have done this because it is quite clear that this category picks up tiny building blocks that do not consist of 'true' buildings in the sense in which we regard them, due primarily to the way the data set is formed. We might even argue that we should exclude the next two categories for the same reasons but because so many blocks are included in these groups, we have retained them, working all our analysis through for the nine remaining bands in the first instance. What we see from Table 5 is that all the results for both the domestic and non-domestic categories cluster around the standard allometric relation in which the coefficient is two-thirds, implying that wall area increases in the manner implicit in Euclidean scaling of building volume. This in turn suggests that wall area does not increase faster than the linear measure of increase in the size of a block, and thus no apparent deformation of shape occurs. There is no real difference between the domestic and non-domestic building stock in this respect, for the biggest differences in the allometric coefficients α are between boroughs with largest value being 0.682 for the domestic stock in Tower Hamlets and the lowest being 0.578 for the non-domestic in the City. The coefficient for all the domestic is 0.646 compared with 0.651 for the non-domestic and for all buildings the coefficient is 0.642. In every case in Table 5, the proportion of the variances explained r^2 are greater than 0.985; for all domestic, it is 0.991, for non-domestic it is 0.995 and for all buildings it is 0.993. The values of these coefficients are all significantly different from zero but in most cases they are *not* significantly different from 0.66.

All this might seem quite surprising in the light of Bon's (1973) and our own results on building footprint areas for the same data set (Batty *et al.*, 2008). However, we

Table 5 Allometric coefficients between wall area and building volume

Borough	Domestic	Non-domestic
Westminster	0.609	0.640
City	0.610	0.578
Hackney	0.689	0.675
Islington	0.616	0.657
Tower Hamlets	0.682	0.648
Camden	0.634	0.644
All London	0.646	0.651
Domestic and non-domestic all London	0.642	

know these results must be biased from our earlier analysis, but there is still a very small probability that a lack of deformation of the geometry of buildings to produce the positive allometry required to increase surface area faster than the Euclidean norm, exists because of complications due to construction and lighting. To an extent, this is also borne out by the lower coefficients for the City for the non-domestic stock which imply that as buildings get bigger, their surface area increases less rapidly than their linear measure; proportionately the wall area gets less than might be expected from geometric considerations.

Nevertheless, analysis of the data set presented here is a first pass at an extremely complicated process of extracting building volumes and surface areas from polygonal data integrated from two different sources which in no way are coordinated, and which contain many – perhaps up to 1 million or more – volumetric structures that are not 'true buildings'. Our next step is to rework the regression analysis on individual building volumes, building footprints and wall areas although there are especially difficult issues in associating exactly and unambiguously wall area with volume from the polygonal data set that we have. We need to either exclude small blocks altogether or develop our algorithms further for identifying how these blocks are part of their parent buildings and this will involve us in a serious extension of this analysis to deal with polygonal pattern recognition in the building stock.

Figure 12 shows graphs of the allometric relations for (a) all domestic, (b) all non-domestic and (c) all buildings by the ten classes. Here is easy to see that the first class introduces bias in that it obviously departs from the inherent linearity of these relations. In fact, although all our statistical tests indicate that these relations are strongly linear, there is slight detection of non-linearity in that as we reduce the number of observations systematically by taking out the first, second, third, and so on classes, the regression lines become slightly steeper, that is the allometric coefficients become larger. When we do this retaining just the top five observations, our coefficients for the domestic, non-domestic and all stock change from 0.646, 0.651 and 0.642 to 0.741, 0.715 and 0.720, somewhat turning the overall results on their head in that the larger classes seem to imply much greater distortion in building shape than the overall analysis. This clearly needs further investigation. We have plotted these changing coefficients for the reduced data sets in Figure 12(d).

We can get some sense of what this data set might yield with respect to the true allometric coefficients by assuming that the relation $W \sim V^{\alpha}$ can be fitted exactly to each value of wall area and volume that we have computed for each size band. This assumes

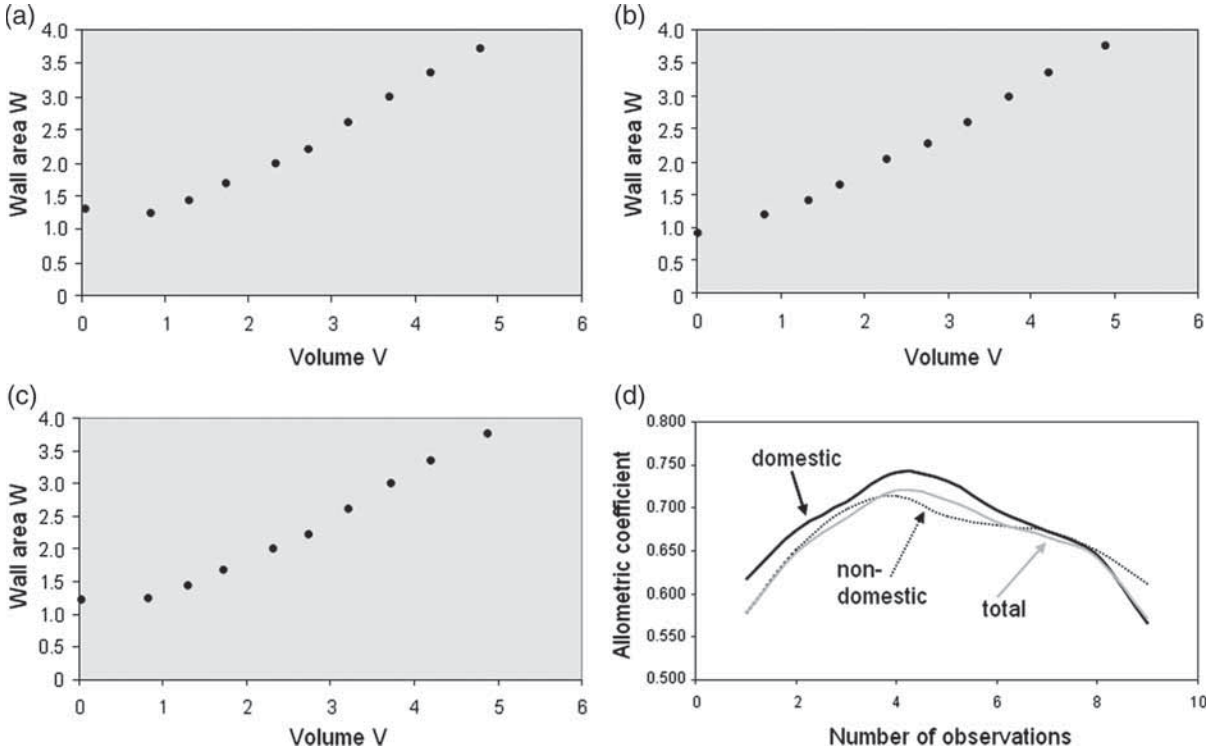

Figure 12 Allometric relations for domestic, non-domestic and all building blocks

that the relation is exact, that is, that $W = V^{\alpha}$ and that there is no scaling constant, or rather we assume a constant of proportionality of unity. If this is the case, then the allometric coefficient for each band can be calculated directly as $\alpha = \log W/\log V$. We can do this for each band in the entire data set, for the ten size classes in each of the six boroughs for the domestic and non-domestic stock and then for the total domestic and non-domestic and then for the total stock. What we find is that for the domestic stock, the allometric coefficients progressively fall as the size associated with each band increases. When we reach the fifth band, the coefficients all fall below 1 and in the highest bands they vary from 0.766 to 0.815, very close to Bon's (1973) value of 0.77. For the non-domestic data, the range for the largest band is 0.764 to 0.776, even closer to Bon's figure, remarkably so.

If we add all the domestic, and then the non-domestic, in the largest band, the coefficients are 0.767 and 0.778. For the entire stock the coefficient rises to 0.857 in the largest band which is clearly an artefact of the way these two distributions are added together. To illustrate the power of this analysis in detecting that the data does display allometry consistent with Bon's (1973) earlier analysis and considerable deformation in building shape as has been argued by Brown and Steadman (1991) many times elsewhere, we have plotted the distributions of these coefficients in Figure 13 where is extremely clear that consistent coefficients only

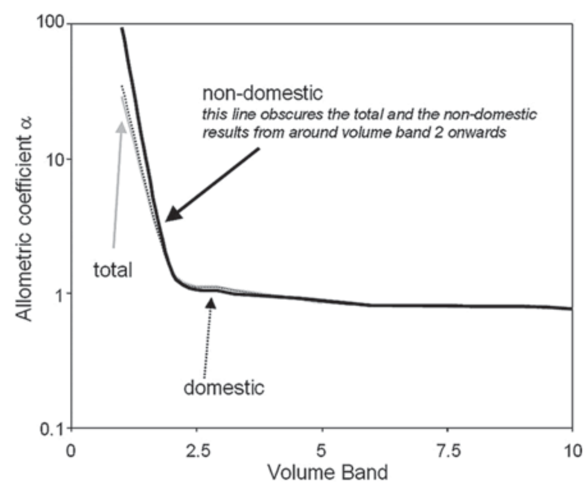

Figure 13 Distribution of individual allometric coefficients for all size bands across all boroughs

begin to appear once the smallest size bands are excluded which is tantamount to excluding the smallest blocks which are disconnected from their parent buildings referred to earlier.

Conclusions

It was shown that when very large numbers of buildings – either domestic or non-domestic – are considered in the aggregate, then their mean plan depths as measured by the ratio V/W are consistent with previous measurements on smaller samples of such buildings. This paper has also demonstrated that when the building blocks that make up the London sample are analysed within their separate size bands, they show allometric relationships between volume and surface of the kind demonstrated by Bon (1973), but only in the larger bands. This is because we are examining in London a dense urban fabric in which many blocks are contiguous. By comparison, Bon measured just detached dwellings. Furthermore, what might on architectural or constructional grounds be termed single 'buildings' can, in the present data, be broken up in a haphazard way – through the process of digitizing their footprints – into many small component parts, for which Bon-type allometry would not be expected to occur. It is only in the larger size bands that the blocks more closely equate to 'buildings', and become partly or wholly detached.

In future work it would be desirable to amalgamate all the smaller blocks into 'buildings', but this is technically not straightforward, and in any case the definition of what should constitute *one* building can be elusive. We have attempted to join the footprint polygons of blocks automatically into something more like building footprints, on the basis of contiguity and data about occupants and addresses, but this has only served to show the difficulties. Another approach, which would certainly be feasible, would be to define detached 'built islands' by taking one block and adding to it progressively all blocks with contiguous footprints until the boundary of the island is reached. But in dense city centres these islands can be large.

Some anomalies in the results here could well be produced by the misclassification of buildings as 'domestic' rather than 'non-domestic', and vice versa; and of course, there are many large buildings that contain both types of use. Cleaning the data of such problems might fine-tune the results. Certainly, the disaggregation of non-domestic into use classes such as office, retail, industrial, etc. should clarify and extend some of the findings.

As for applications, the work has relevance to several areas of building science at a scale larger than that of single structures. The ratio V/W has implications for the capital costs of construction, since it expresses the area of building envelope per unit of accommodation. Building depths are important for energy use, since a distance of 6 m or 7 m from exposed walls is, as was shown, the approximate limit of the passive zone beyond which artificial ventilation and permanent artificial lighting generally become necessary – as formalized in the LT model. The measurements for non-domestic buildings presented here have shown, in effect, the extent of non-passive core space beyond this depth limit. Recent research by Salat (2009) has revealed substantial variations in overall surface-to-volume ratios between different cities, and has shown the impact of the passive-to-non-passive volume ratio on total energy use for heating.

Acknowledgements

The measurements on which this analysis are based were made by Stephen Evans for the EPSRC LUCID Project (the development of a Local Urban Climate model and its application to the Intelligent Development of cities: Grant Number EP/E016375/1) led by Professor Michael Davies at University College London.

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Endnotes

[1]This argument relates to multi-storey buildings. A single-storey block can be made deeper and the centre can be daylit via roof lights – although these are not completely equivalent to windows in functional terms, of course, since they provide views only of the sky.

[2]This argument applies to buildings like apartment blocks with continuous longitudinal circulation routes, but not to blocks broken up, for example, into terrace houses.

[3]The shallow buildings may also be air-conditioned – although they do not absolutely have to be – for reasons such as acoustic insulation from traffic noise.